DEVELOPMENT OF A THEORETICAL MODEL FOR COAL CLEANING PROCESSES

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CERTIFICATE

This is to certify that the present work 'DEVELOPMENT OF A THEORETICAL MODEL FOR COAL CLEANING PROCESSES' by Mr. M. Tamilmani has been carried out under my supervision and has not been submitted elsewhere for a degree.

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CONTENTS

LIST OF TABLES

LIST OF FIGURES

SYNOPSIS

A number of coal cleaning processes which are based on the float-sink principle are in use to clean a wide variety of coal changing both in quality and size consist. For selecting a suitable process for a particular run-of-mine coal and forecasting the cleaning results, one needs performance criterion. Generally distribution (Tromp) curve which is obtained when the per cent of feed reporting to clean coal is plotted against density is used for this purpose. A formal model for coal cleaning processes that will result in distribution curve has been developed. The values of the parameters in the model are estimated using non-linear optimisation techniques namely Box-Complex method and Rosenbrock Hill Climb method. The variation of parameters with feed size gives a better insight into the performance of different coal cleaning equipments.

CHAPTER 1

INTRODUCTION

Coal is the most abundant fossil fuel resource of energy. Raw coal is always associated with impurities. Gradual depletion of the high quality coal seams and increasing use of mechanised mining methods has led to the substantial increase in the impurity content of the raw coal. Frequently the raw coal is not acceptable for power-plants, metallurgical or other uses. A thorough cleaning of coal, to reduce the ash and sulphur impurities is now almost mandatory for coal utilisation. Because of the wide variations in the nature of coal, ash content, size fractions, etc., coal cleaning is performed by various processes. For the selection of the optimum process and its monitoring, for a particular type and size of coal, some performance criterion is needed.

Of the many performance criteria available for this purpose, distribution (Tromp) curve is most commonly used. In it the performance of various coal cleaning machines and the effect of control setting is given interms of error area, probable error etc. The mathematical form of the Tromp curve is identical to the cumulative distribution function. However, the interpretation of the two are different. In particular, the frequency function of Tromp

curve has no physical meaning. Many attempts have been made to describe Tromp curves interms of statistical cumulative distribution functions, including normal, log-normal and Weibull distributions. However, as of now a formal model for coal cleaning that will generate Tromp curve is not available.

In this thesis, we present a mathematical model of the coal cleaning process that generates a separation (Tromp) curve. The model is extended to incorporate the realistic case where the minimum density of coal is greater than zero, maximum coal fraction floated is less than one and minimum coal fraction in sink is greater than zero. It is shown that the model's separation curves are self preserving or self-similar interms of a dimensionless density, $\frac{d}{d_{0.5}}$, inconformity with the results in literature. In the model, the sharpness of separation is quantified by a sharpness index n which is estimated using Box complex technique and Rosenbrock hill climb method. The variation of parameters with feed sizes in five different processes, dense medium cyclone, hydro cyclone, concentrating table, dense medium vessel and baum-jig has been determined and analyzed.

CHAPTER 2

LITERATURE REVIEW

2.1 NEED FOR COAL CLEANING

Coal is a mixture of combustible metamorphosed plant remains that vary in both physical properties and chemical composition. A number of components are present in the coal, depending on the nature of the original plant materials and the degree of coalification. In addition the coal coming from the pits and seams contain a significant amount of impurities, both ash forming mineral matter and sulphur contributing compounds. The origin, composition, properties and the distribution of impurities have been discussed by many authors [1-3]. Shale, Kaolin, sulphide, carbonate and chloride group species are the major mineral impurities. A number of minor minerals, quartz, feldspar, etc., are also present. The impurities are subdivided into 'inherent impurities', which are structurally a part of the coal itself and cannot be separated from it by any mechanical/physical means, and 'segregated impurities', that exist as individual, discrete particles, and can be removed by mechanical coal cleaning.

Yancey and Geer [4] have summarized the ill effects of impurities:

1. An economic loss is manifest in the handling, transportation, and storage of high-ash coal, for the inert impurities must be handled with the coal, yet contribute nothing in its use.

2. In combustion, the ash-forming impurities not only dilute the combustible content of the coal and thus reduce its calorific value but also lower the effective capacity of the burning equipment, reduce the plant flexibility, and impose added expense in handling of ash.

3. The fluctuations in ash content that characterize the high-ash coals render the control of combustion difficult.

4. Excessive ash content impairs the efficiency of combustion and increases the erosion/corrosion of the equipment.

5. High ash content in the coal decreases the yield of coke interms of carbon content or thermal value, decreases the yield of by-products and gives a weaker coke and consequently higher percentage of breeze fraction.

6. Use of high-ash coke in blast furnace leads to an increase in the consumption of coke, requires more limestone for flux, reduces furnace capacity, and makes the temperature control more difficult.

7. Sulphur, the second impurity after ash is a deleterious impurity in virtually all the metallurgical products. For this reason, high - sulphur coal is unsuited for smelting

or heat-treating processes. For example, in blast furnace operation, the penalities exerted by high-sulphur coke are added slag volume, increased coke consumption, and greater difficulty in furnace control.

8. When coals containing more than about 0.05 per cent sodium chloride are coked, the refractory lining of the coke ovens suffers serious corrosion.

The said ill effects can be minimized to a lesser or greater extend by coal cleaning which is defined as a physical and/or chemical process for removing high-ash impurities including pyritic sulphur from raw coal. The attributes of cleaned coal are greater uniformity in composition and reduced impurity content. Zachar and Gilbert [5] have discussed the economics of coal preparation in detail. According to them, the removal of free impurities from raw coal to upgrade the product inorder to make it saleable is an expensive process employed only through necessity:

1. to increase its net income per ton of product, and
2. to provide a steady outlet for its products.

Mechanization and the use of continuous mining equipment invariably produce raw coal containing more impurities. Coal preparation therefore permits rapid, full-seam mining. Also the amount of impurities in fine coal tends to be high, which again calls for appropriate coal cleaning facilities.

2.2 COAL CLEANING PROCESSES

Most of the coal cleaning processes depend on the density difference between coal and its associated impurities. Coal ranges in apparent density from 1.2 to 1.7, depending on its rank, moisture content, and percentage of ash. Shale, clay, and sandstone range from about 2.0 to 2.6 in density, depending on their degree of purity; pyrite ranges from 2.4 to 4.9; and calcite and gypsum have a density of 2.7 and 2.3, respectively. All removable impurities associated with coals are higher in density than the coal. The two major difficulties, that have influenced the development of a number of coal cleaning processes are:

1. The raw coal contains middlings of a wide range of intermediate ash contents; and
2. The relative behaviour of the coal particles of different sizes.

The classification of the coal cleaning equipments and the principles underlying them have been dealt by many authors [6-9]. A simple classification of the more important wet, gravity concentration methods/equipments is:

1. Dense medium vessels
2. Jig washers
3. Dense medium cyclones
4. Hydrocyclone
5. Concentrating Tables.

Dense medium vessels and jig washers are normally used for the concentration of coarse coal. Palowitch and Deurbrouck [9a] have described the dense medium process, the separator design, the various kinds of equipment available and the advantages of dense medium processes over other coal cleaning processes. A major portion of the coals is washed by dense medium processes, because of the following characteristics of this process:

1. Ability to handle a wide range of sizes
2. Ability to make sharp separations at any density within the normal range, even in the presence of high percentages of the feed in the range of ± 0.1 density units.

A major disadvantage is that the fine sizes cannot be treated, because they decrease the density of the medium and cannot be recovered from the sand (suspension material) by screening.

The unit operation of jigging, the different types of jigs used and the design of Baum-jig have been given by Lovell [9b]. Jigs are employed to clean either sized or unsized coal; in a few instances they have been used to treat in one operation all sizes from dust upto 6 or even 8 inches. When such a wide size range is employed, the cleaned product may meet the market requirements, but the efficiency with which some of the individual sizes are treated, is

necessarily limited. Jig cleaning efficiency tends to decrease with the smaller sizes and tends to clean best at higher densities as size decreases. When a very broad size range is cleaned by a jig, the efficiency of separation decreases with a decrease in average size. According to Yancey and Geer, the recovery efficiency was nearly as good as attained with heavy media process.

The cyclones and concentrating tables are mainly used for cleaning of fine coal. Sokaski, Geer and Yancey [9c] have described the theory, flowsheet, description and performance of dense medium cyclones and hydrocyclone. In general, the dense medium cyclone effects a sharper separation between coal and impurity than can be obtained in other types of cleaners handling the same size range.

As of now, cyclones are used to treat a comparatively narrow size range, typically 3/8 or 1/4 inch to 1/2 mm. Therefore, little information on how various sizes behave in the cyclone is available. It would be however expected that coarse coal would be separated more sharply than fine coal. When cleaning is performed at a high density setting, the superior sharpness of separation accomplished in the cyclone does not greatly improve the recovery efficiency and the economics of the operation. However, if the separation must be carried out at a low density, cyclone alone can produce washed coal of required sulphur or ash content, at higher efficiency.

The separations obtained in a hydrocyclone are not nearly as sharp as those that are characteristic of the dense medium cyclone. Consequently hydrocyclones are not applicable for difficult to clean coals or separation at low density. The hydrocyclone could especially be suited for treating minus 28-mesh coal, more so if the coal is not amenable to flotation. The hydrocyclone is reported to be superior to flotation for lowering the sulphur content of the washed coal, if iron pyrite is present in the feed.

Deurbrouck and Palowitch [9d] have described the principle of tabling processes, operating conditions and their performance characteristics. The concentrating table cleans the fine particles much more efficiently than either hydrocyclones or the fine coal jig (feldspar jig). However, minus 200-mesh material was found not responding to upgrading by tabling.

Using the flow-sheet of Bhojudih washery, located in the eastern part of Jharia coal field, Gokale and Rao [10] have illustrated, how the different cleaning processes are combined in a coal washery. Here coal is crushed to minus 3-inch size and screened on 1-inch screen. The minus 3-inch plus 1-inch fraction is cleaned by heavy media separation and the minus 1-inch fraction is washed in baum jigs. The sinks of heavy media and the jigs are then crushed to minus 1/2 - inch and washed in dense media cyclones.

Based on the data for various schemes of washing in vogue at different washeries as compiled by Chakravarti and Moritra, Gokale and Rao observe the following points:

1. The accuracy of separation at any predetermined density within the range of 1.25 - 2.50 and continuous maintenance of preselected value of density within ± 0.005 of its value, are the salient features of dense medium processes. The additional qualifications of heavy media separation are its ability to separate at one or more predetermined densities and its capacity to treat a larger size range of coal, besides its low cost, flexibility, and accuracy of control.

2. Baum-jigs are employed where initial separation of shales from coals is required. The separation is effected at a density between 1.70 and 1.80.

3. High ash content and inferior coals require the flotation treatment.

4. The dense medium cyclones can take care of the finer size (less than 1.25 cms in size) coals of difficult washing characteristics, with an optimum recovery.

2.3 THE PERFORMANCE CRITERIA

It is evident that the selection of the best process flow for a given run-of-mine coal requires a careful study wherein many conflicting factors must be weighed. The cost of a detailed investigation is well repaid capital investment,

in higher recoveries, in flexibility, and in the ease of operation and maintenance. In order to compare the different cleaning operations, the performance of coal cleaning equipment or kinds of coal, and to forecast the cleaning results we need a suitable performance criterion.

2.4 TYPES OF PERFORMANCE CRITERIA

Geer and Yancey [11] have classified the various performance criteria according to the degree to which they are dependent on the density composition of the raw coal, namely directly dependent, indirectly dependent and independent criteria.

The yield and ash content of the washed coal are two performance factors of direct and immediate interest. These two factors are directly dependent on thedensity composition of the raw coal treated and the functioning of the equipment. The continuous variation in the amount of impurity in the raw coal makes it impossible to tell whether a reduction in yield or an increase in the ash content results from any malfunction or misadjustment of the cleaning equipment. As such these performance factors are not suitable for comparing the performance of different plants, because no two plant feeds are the same.

The important performance criteria, that are indirectly dependent on the density composition of raw coal are:

1. Efficiency formulas
2. Misplaced material concept, and
3. Ash error and Yield error.

The well known efficiency formula given by Fraser and Yancey is,

$$\text{Efficiency} = \frac{\text{Yield of washed coal}}{\text{Yield of float coal of same ash content}} \times 100$$

This type of efficiency formula has a drawback of giving a value of 100, when no impurity is removed. In pneumatic cleaning equipments impurities finer than 28-mesh size are not removed. So for such fine size raw coal, the efficiency becomes 100. To check such drawbacks, ash reduction must be considered along with efficiency. Satisfactory performance then necessiates both high efficiency and the required ash content in the washed coal. Also to account for the coal that is freed by degradation during the washing process, one has to use calculated feed in the efficiency formulas.

'Misplaced material' is the sum of the sink in the washed coal and the float in the refuse, generally expressed as a percentage of the raw coal. Although the 'misplaced material' concept is simple to understand, it does not take into account the quality of the misplaced material. It does not relate directly to the all important factors of yield and ash content.

Ash error is the numerical difference between the actual and theoretical ash contents of washed coal at the yield of washed coal obtained. It is a direct measurement of the impairment on account of ash content. Yield error is the difference between the yield of coal actually obtained and the theoretical yield at the ash content of the washed coal. It expresses the loss in yield. The authors [11] have given the general relationship between efficiency, misplaced material, ash error and yield error, in the form of a tabulated data. The restrictions on the applicability of these directly and indirectly dependent criteria are stated as follows:

'Equipments treating coals of substantially similar density compositions or operating at substantially same densities of separation alone can be compared on the basis of these criteria.'

2.5 DISTRIBUTION OR SEPARATION (TROMP) CURVE

The type of separation between coal and impurity obtained in actual practice is illustrated graphically in Fig. 2.1. Coal of low density and impurity of high density report largely or entirely to their proper products. As the density of separation is approached, more and more material reports to the improper product. The infinitesimal increment of density that is divided equally between the washed coal and refuse is defined as the density of separation.

Tromp observed that the curve in Fig.2.1, which has the shape of a crude Gaussian error distribution, is governed by the laws of probability. Also, if the percentage of recoveries of washed coal are plotted against the mean densities of the density fraction, the type of curve in Figure 2.2 is obtained. The curve in Figure 2.2 has many names such as Tromp curve, distribution curve, separation and recovery curve. The distribution curve in Figure 2.2 can also be obtained by rotating the left-hand limb of curve in Figure 2.1. by 180°, about the 50 percentage ordinate.

When the distribution curve is plotted on either log probability or arithmatic probability grid, one will occasionally get a straight line plot. The advantages inherent in a straight-line plot of this kind are:

1. The slope of the line is a measure of sharpness of separation. Steeper the slope, sharper is the classification.

2. The slope and the specific gravity of separation combine to characterize the complete curve.

However, in practice, many distribution curves do not exhibit a straight line throughout the full range on log normal or normal plot.

As stressed by Gottfried [12], distribution curve is a plot of the weight per cent of feed reporting to clean coal, f(d), as a function of density, d. The yield of clean coal, Y is,

$$Y = \frac{\int_0^{d_1} f(d)\, m(d)\, dd}{\int_0^{d_1} m(d)\, dd}$$

Where m(d) represents the feed rate of material having density d and d_1 represents the highest density value of the feed material.

The density of the material in the raw feed that is divided equally between the clean coal and refuse is defined as the density of separation, d_s. In otherwords, it is the value of density corresponding to f(d) = 50. By making appropriate physical adjustments in a coal washing machine, the value of the density of separation d_s can be increased or decreased, thus shifting the entire distribution curve to the right or left.

Gottfried and Jacobsen [13] have discussed the distribution curve for a theoretically perfect separation and an actual separation case as in Figure 2.2. The shaded area, called error area, in Figure 2.2 is a measure of the difference between the actual and theoretically perfect separation. The error area approaches zero as the actual distribution curve approaches the theoretical, perfect classification. Error area is dimensionless and ranges from as low as about 10 for good dense medium operation to over 100 for some inferior table and jig operation.

Another measure of the distribution curve called 'Probable error' also represent the sharpness with which the coal and impurity are separated. The quantity is equal to one-half the density difference between the 25 and 75 per cent ordinate values on the curve. The steeper the distribution curve, the lower the probable error. Figure 2.2 illustrates the relationship between probable error and the slope of the distribution curve.

Geer and Yancey [11] have compared the distribution data for four dense medium plants and concluded that within reasonable limits, the separation between coal and impurity is independent of the density composition of the feed. For other processes also the independent nature of the distribution curve has been established.

The size composition of the raw coal influences the distribution curve. In dense medium vessel and dense medium cyclone, coarse sizes are separated somewhat more sharply than the fine sizes. In jigs and tables this dependence is even more pronounced. The distribution curves for the coarse, intermediate and fine sizes of coal, cleaned on a concentrating table have been given by Geer and Yancey [11].

2.6 GENERALIZED (SELF PRESERVING) DISTRIBUTION CURVE:

For a given coal-cleaning equipment and a given size distribution of coal, different distribution curves as in Figure 2.3 are obtained for different settings of density of

separation. The shape of these different distribution curves are similar, because of the following factors.

For a given size distribution of coal and machine

1. Tromp curve is independent of density distribution of coal and
2. Tromp curve is independent of density of separation, $d_{0.5}$ setting.

A method for combining these different distribution curve into one generalized or self preserving curve, which will apply for any density of separation within the range of original data, is presented by Gottfried and Jacobsen [13]. Here the weight per cent of feed reporting to clean coal; $f(\bar{d})$, is plotted against the reduced density, $\bar{d}$. The reduced dimensionless density of separation is defined as the ratio of density to the density of separation.

i.e. $$\bar{d} = \frac{d}{d_{0.5}}$$

The resulting distribution curve is independent of the density of separation. They have shown, how the several conventional distribution curves of given coal-cleaning equipment treating particular size distribution of coal, can be represented by a single generalized distribution curve graphically. The significance of the generalized distribution curve is that it can be used with any density of separation, not only the density of separation corresponding to the given data.

The effect of size on the generalized distribution curve, the uses and range of applicability of generalized distribution curve have been well dealt by Gottfried and Jacobsen [13]. Figure 2.3 shows conventional distribution curves and Figure 2.4, the resulting generalized distribution curve.

2.7 MATHEMATICAL FORM OF DISTRIBUTION CURVE

The various attempts at mathematical representation of distribution curve have been dealt by Gottfried [12]. Tromp recognized the similarity between the distribution curve obtained for a coal cleaning device and a normal (Gaussian) distribution function. This normal distribution function is not suitable because.

1. The cumulative normal distribution function cannot be expressed in a closed form (that is, in terms of a simple algebric formula). This is also true for log normal distribution.

2. Many coal -cleaning devices exhibit asymmetric distribution curves, whereas the normal distribution is perfectly symmetrical.

Gottfried [12] has suggested that the weibull function, an exponential type distribution function may turn out to be a better candidate for a mathematical representation of the generalized distribution curves.

1. Weibull function has several interesting characteristics, including a closed-form representation for the

cumulative (integrated) function.

2. With proper choice of parameters, it can provide a reasonably near closed-form approximation to the cumulative normal distribution function.

3. It can also be used to represent asymmetric distribution curves.

The cumulative (integrated) distribution form of Weibull frequency function is,

$$f(\bar{d}) = 100 \exp [-(\bar{d} - \bar{d}_1)^a/b] \qquad (2.1)$$

Where $\bar{d}$ is the reduced density, $f(\bar{d})$ is the per cent of feed of reduced density $\bar{d}$ reporting to clean coal. a and b are constants. Equation (2.1) will apprpximate the generalized distribution curve. Since in actual practice of coal-cleaning, the maximum floated coal may not be 100 per cent but something like 98 or 99 per cent and the minimum floated may not be zero but as high as 30 or 40 per cent, equation (2.1) is modified as follows [12]:

$$f(\bar{d}) = 100 [f_o + c \exp -(\bar{d} - \bar{d}_1)^a/b \;] \qquad (2.2)$$

Where f_o+c is always less than or equal to 1. Applying the condition that $f(\bar{d}) = 50$, when $\bar{d} = 1$,

$$\bar{d}_1 = 1 - [b \log_e \frac{c}{0.5 - f_o}]^{1/a} \qquad (2.3)$$

Hence, the number of independent parameters is 4, that is f_o, a, b and c.

Equation (2.2) can be used to represent the generalized distribution curve, provided a,b,c and f_o are determined for a given set of distribution data. By nonlinear regression technique Gottfried [12] computed the values of a,b,c and f_o and tabulated them for several different cleaning vessels and a number of feed size fractions.

Although the fitted weibull functions represent the experimental data reasonably well, it has a number of disadvantages.

1. Many of the curves broke too sharply at the extreme points, that is at the knees of the curves.

2. The fitted curve often fall short of spanning the complete range namely 0-100 per cent. The disparity between the analytical and the tabular forms of the distribution data is quite severe in some curves.

3. The choice of the Weibull distribution is based on an adhoc assumption, and it is difficult to relate its 4 parameters to the underlying physical process of coal cleaning.

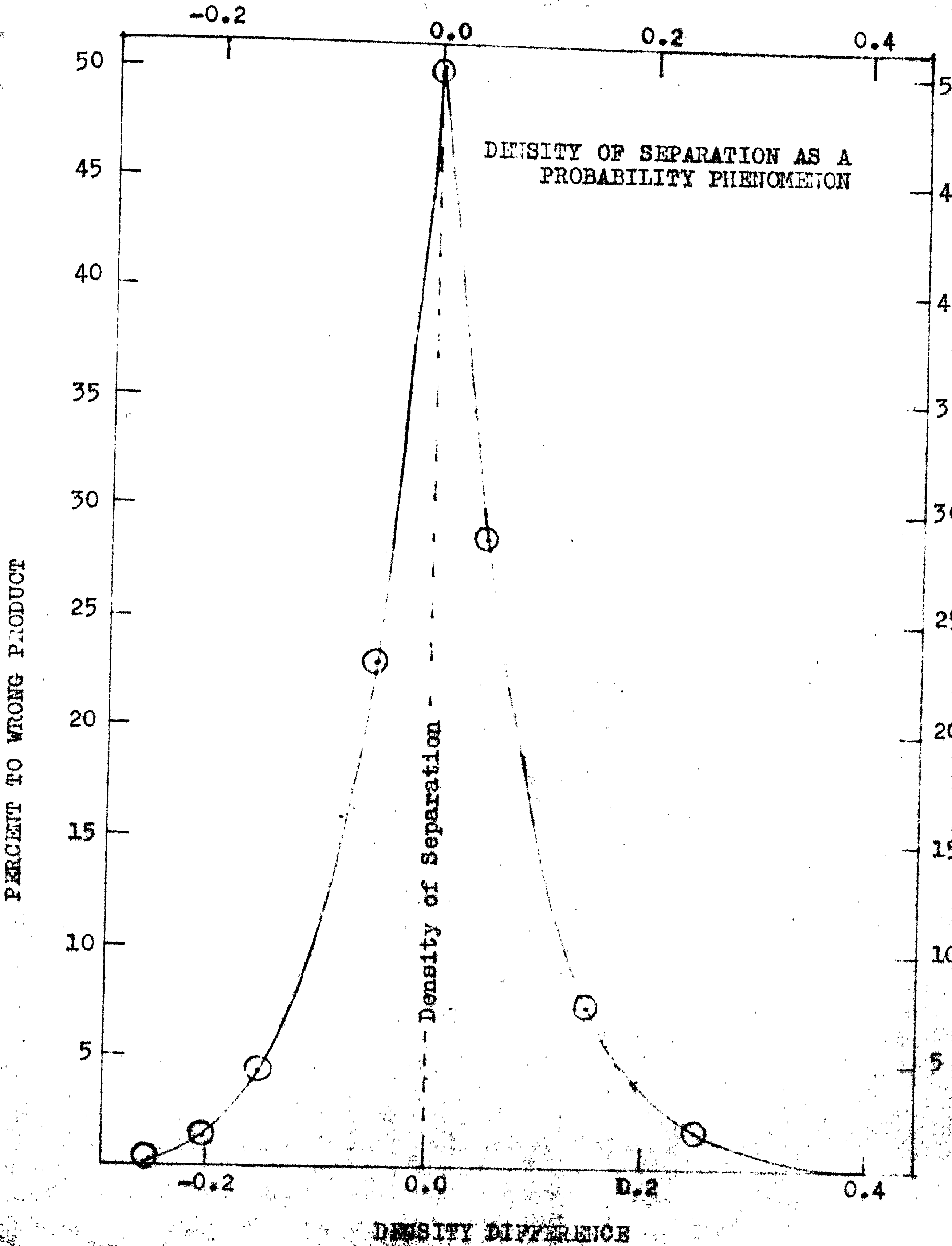
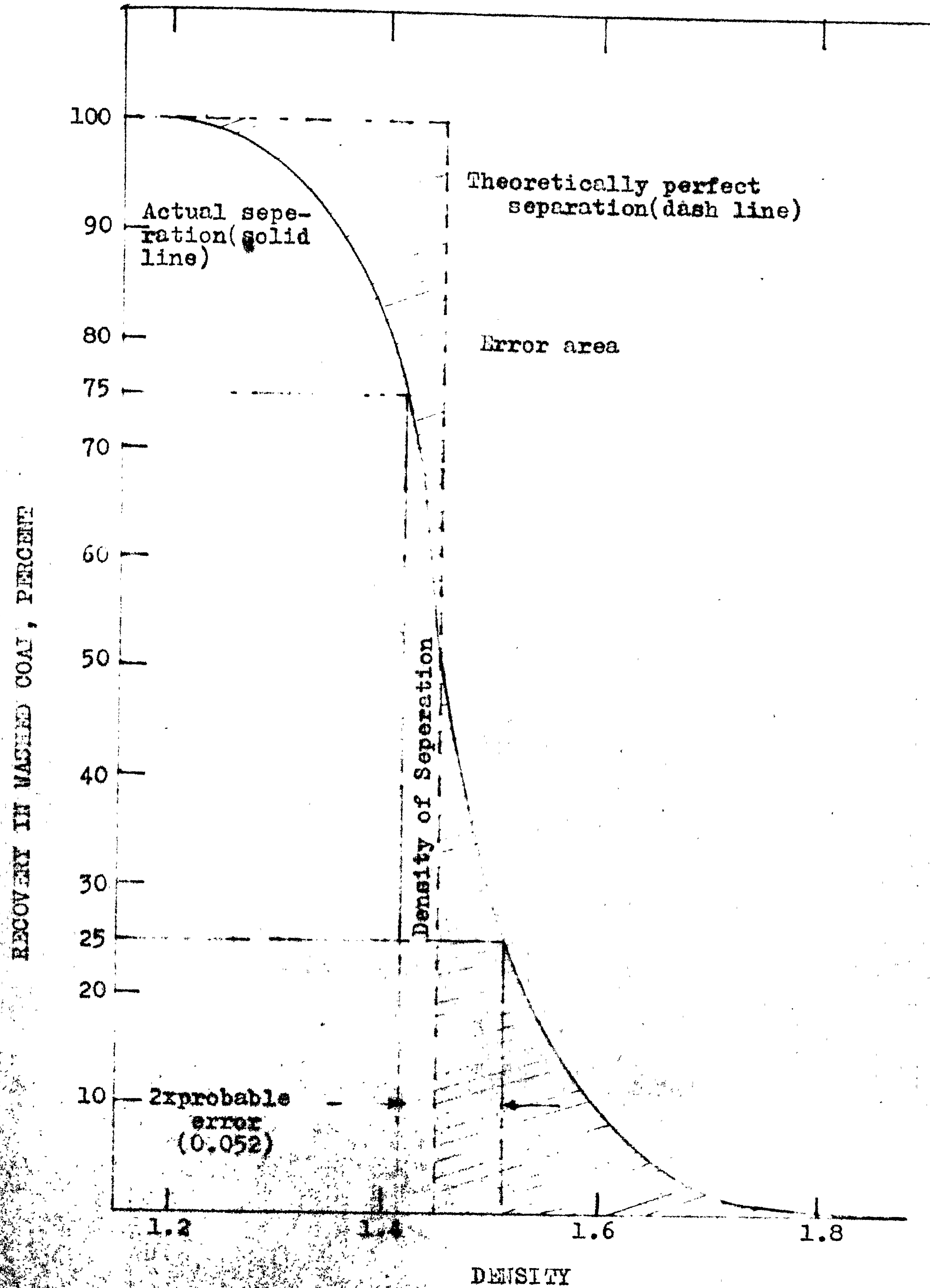
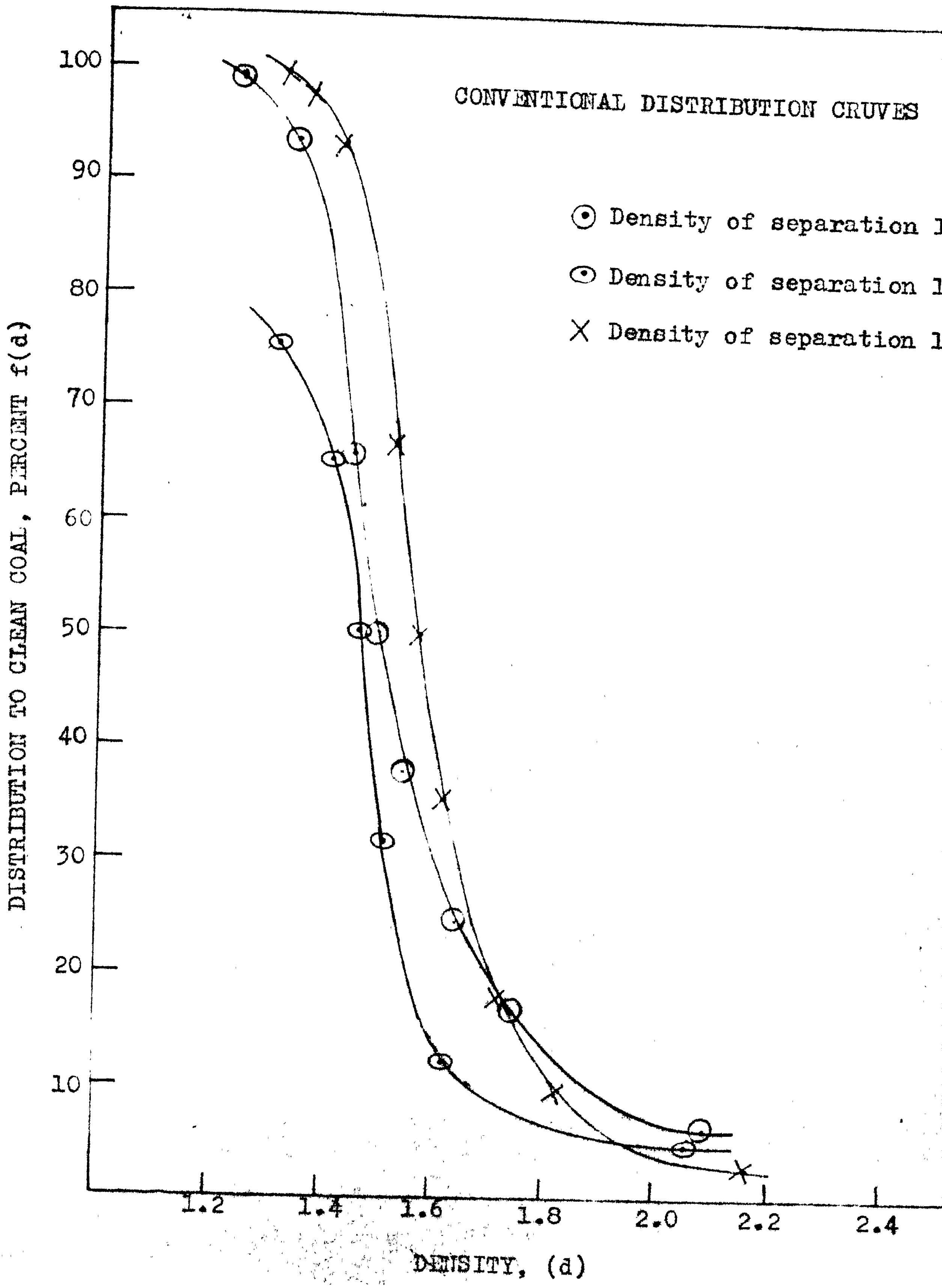

-0.2
0.0
0.2
0.4
DENSITY OF SEPARATION AS A PROBABILITY PHENOMENON
50
45
40
35
30
25
20
15
10
5
PERCENT TO WRONG PRODUCT
Density of Separation
-0.2
0.0
0.2
0.4
DENSITY DIFFERENCE

FIGURE 2.1



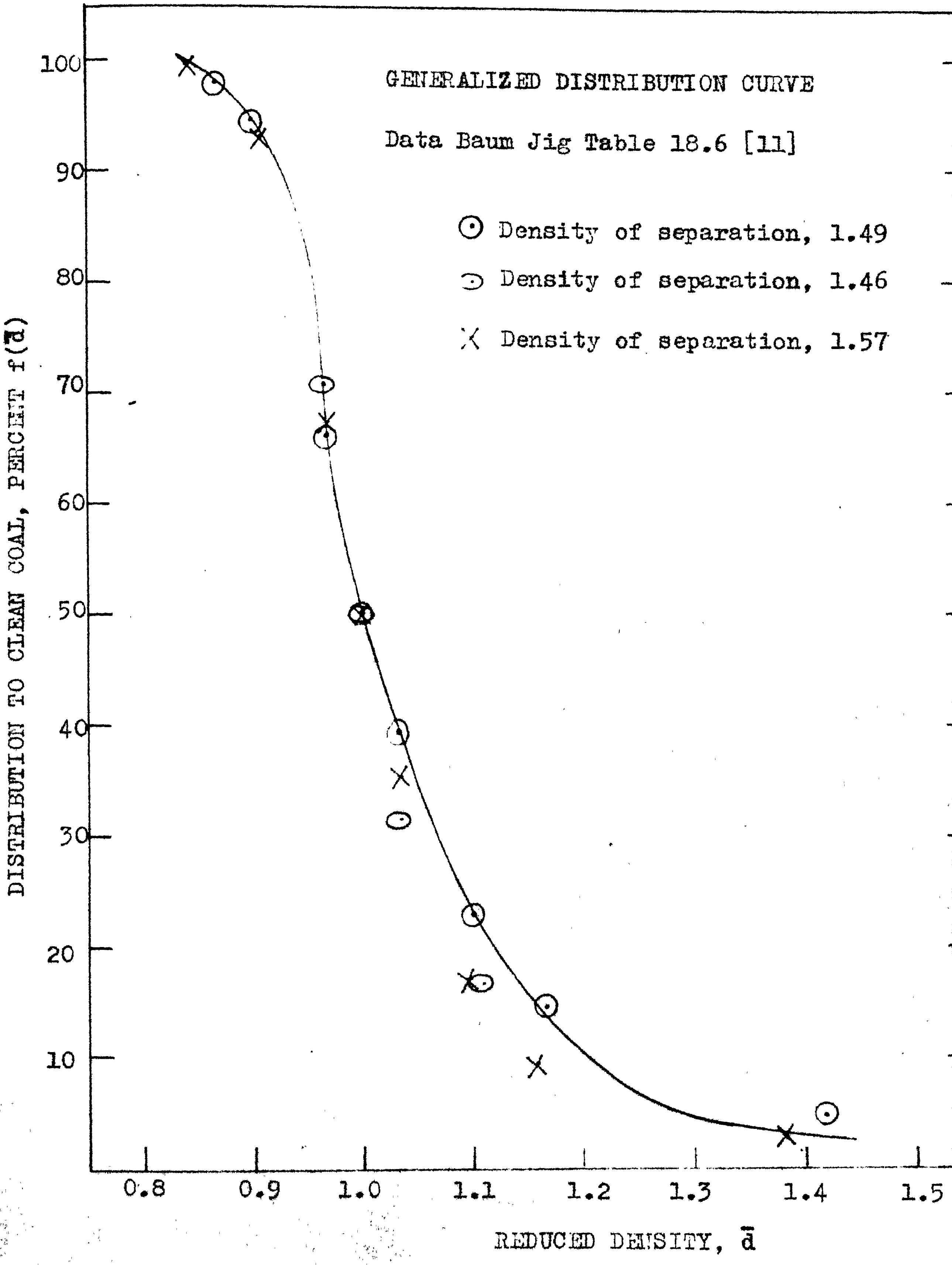
100
90
80
75
70
60
50
40
30
25
20
10
RECOVERY IN WASHED COAL, PERCENT
Actual sepe-
ration(solid
line)
Theoretically perfect
separation(dash line)
Error area
Density of Seperation
2xprobable
error
(0.052)
1.2
1.4
1.6
1.8
DENSITY

FIGURE 2.2



CONVENTIONAL DISTRIBUTION CRUVES
Density of separation 1
Density of separation 1
Density of separation 1
DISTRIBUTION TO CLEAN COAL, PERCENT f(d)
100
90
80
70
60
50
40
30
20
10
1.2
1.4
1.6
1.8
2.0
2.2
2.4
DENSITY, (d)

FIGURE 2.3



GENERALIZED DISTRIBUTION CURVE
Data Baum Jig Table 18.6 [11]
Density of separation, 1.49
Density of separation, 1.46
Density of separation, 1.57
DISTRIBUTION TO CLEAN COAL, PERCENT f(d̄)
100
90
80
70
60
50
40
30
20
10
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
REDUCED DENSITY, d̄

FIGURE 2.4

CHAPTER 3

OBJECTIVE

From the previous chapter on the literature review of this subject, we conclude that there is at present no formal model on coal cleaning processes that results in Tromp curve. Our objective in this work is to develop a model for a class of coal cleaning processes from a set of phenomenological concepts of the cleaning machines. We show that the model generates a distribution curve which is self-similar. That is, the separation is independent of the density composition of the raw coal feed and reduced density of separation, within reasonable limits. Further, the model is tested on computer using the rather extensive data provided by Geer and Yancey [11] and Gottfried and Jacobsen [13]. The sharpness index is estimated and analysed for five kinds of coal washing machines and a broad range of coal size fractions.

CHAPTER 4

MODEL

Most coal cleaning equipments employ a float-sink principle, based on the density difference between coal and associated impurities. Thus a typical piece of coal cleaning equipment will separate a given feed into a low density clean coal product and a high density refuse. The schematic diagram of a float-sink process is given in Figure 4.1.

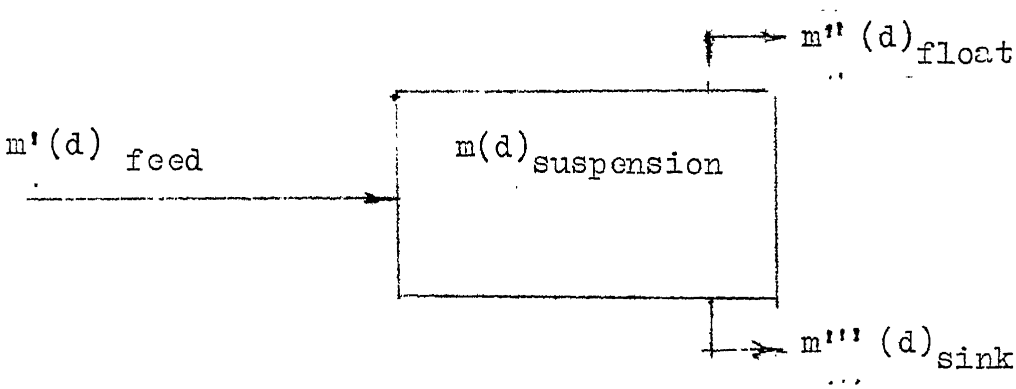


Figure 4.1

Let $\dot{m}'(d)$ be the mass flow rate of the raw coal of density d that is fed into the cleaning equipment, $\dot{m}''(d)$ be the mass flow rate of the washed coal, $\dot{m}'''(d)$ be the mass flow rate of the impurity and m(d) be the steady state mass of coal in the suspension in the machine.

Since the rate of flotation depends on (1) mass of coal that is present in the suspension (2) density of coal in the suspension and machine characteristics, we can write,

Rate of float α $m(d)$

α d^a

Therefore, Rate of float = $\bar{K'}\ d^a\ m(d)$

Where a is some exponent and $\bar{K'}$ is the machine constant.

Similar rate of sinking of impure material is,

Rate of sink α $m(d)$

α d^b

or, Rate of sink $= \bar{K''}\ m(d)\ d^b$

Where $\bar{K''}$ is also a machine constant.

At steady state, by mass balance,

Rate of feed = Rate of float + Rate of sink

i.e. $m'(d) = \bar{K'}\ d^a\ m(d) + \bar{K''}\ d^b\ m(d)$

i.e. $m'(d) - \bar{K'}\ d^a\ m(d) - \bar{K''}\ d^b\ m(d) = 0$ (4.1)

$$\text{Therefore } m(d) = \frac{m'(d)_{feed}}{\bar{K'} d^a + \bar{K''}\ d^b} \qquad (4.2)$$

$$\text{Since } m''(d)_{float} = \bar{K'}\ d^a\ m(d) \qquad (4.3)$$

Substitution of equation (4.2) gives,

$$m''(d)_{float} = \frac{m'(d)_{feed}}{\bar{K'}\ d^a + \bar{K''}\ d^b}\ \bar{K'}\ d^a \qquad (4.4)$$

Mass fraction recovered is,

$$f(d) = \frac{m''(d)_{float}}{m'(d)_{feed}} \qquad (4.5)$$

Substituting the values of $m''(d)_{float}$

$$f(d) = \frac{K' d^a}{K' d^a + K'' d^b} = \frac{1}{1 + K d^n} \qquad (4.6)$$

where,

$$K = \frac{K''}{K'} \qquad \text{and } n = b-a \qquad (4.7)$$

We know at the split point, that is at the density of separation $d_{0.5}$, 50 per cent or 0.5 fraction will be recovered. That is $f(d_{0.5}) = 0.5$

Substituting this value of $f(d_{0.5})$ in the equation (4.6) we get,

$$0.5 = \frac{1}{1 + K d_{0.5}^n} \qquad (4.8)$$

Therefore $$K = \frac{1}{d_{0.5}^n} \qquad (4.9)$$

Substituting this value of K in equation (4.6) leads to

$$f(d) = \frac{1}{1 + \left(\frac{d}{d_{0.5}}\right)^n} \qquad (4.10)$$

Let a dimensionless density $\bar{d}$ be defined as $\bar{d} = \frac{d}{d_{0.5}}$, then equation (4.10) becomes,

$$f(\bar{d}) = \frac{1}{1 + \bar{d}^n} \qquad (4.11)$$

This is the basic model of float-sink processes. Here $f(\bar{d})$ is the fraction that is recovered, which is a function of some dimensionless density $\bar{d}$ and n is the sharpness index.

If the density of coal considered is zero, the reduced density $\bar{d}$ is zero and the fraction recovered becomes 1; that

is all the feed will be floated. If the density considered is infinity, $\bar{d}$ is also infinity and the fraction recovered becomes zero; that is nothing of infinite density will be floated. If the density of coal under consideration (d) is the same as that of density of separation, that is $d_{0.5}$, then $\bar{d} = \frac{d_{0.5}}{d_{0.5}} = 1$, and the fraction recovered will be 0.5.

Using the basic equation (4.11) of float-sink process, the nature of the distribution curves for various values of the sharpness index, n are determined, using the programme in Appendix 3. The computed values of f(d) for different sharpness index, n are given in the same appendix. The same results have also been shown graphically in Figure 4.2. As the sharpness index, n, value increases, the distribution curve becomes steeper and approaches the ideal or theoretically perfect separation.

Note that n, the index of separation, is always greater than zero. For the case, n is infinity:

(i) $d < d_{0.5}$, $\bar{d} < 1$ and $f(\bar{d}) = 1$

(ii) $d > d_{0.5}$, $\bar{d} > 1$ and $f(\bar{d}) = 0$

(iii) $d = d_{0.5}$, $\bar{d} = 1$ and $f(\bar{d}) = 0.5$

This is the ideal case for perfect separation shown in Figure 2.2.

The above basic model is based on:

(1) Minimum density is zero,

(2) Maximum float is 1, and

(3) Minimum float is zero.

But in actual practice the minimum density may have some value, d_1, greater than zero, the maximum float f_u, may be less than 1, and the minimum float f_1, greater than zero (See Figure 4.4).

We now proceed to modify our model to extend its validity. First, still assuming $f_u = 1$, and $f_1 = 0$, but $d_1 > 0$, we developed a 2-parameter model in the following manner.

We modify equation (4.1) as,

$$m'(d)_{feed} - K'(d-d_1)^a\, m(d) - K''\; m(d)\; (d-d_1)^b = 0 \tag{4.12}$$

That is instead of considering d alone, we consider $(d-d_1)$ Hence equation (4.6) becomes

$$f(d) = \frac{1}{(1+K(d - d_1)^n)} \tag{4.13}$$

When $f(d) = 0.5$ and $d = d_{0.5}$

$$0.5 = \frac{1}{(1+K(d_{0.5}-d_1)^n)} \tag{4.14}$$

and equation (4.10) becomes

$$f(d) = \frac{1}{1 + \left(\frac{d-d_1}{d_{0.5}-d_1}\right)^n} \tag{4.15}$$

or

$$f(\bar{d}) = \frac{1}{1+\left(\frac{\bar{d} - \bar{d}_1}{1 - \bar{d}_1}\right)^n} \tag{4.16}$$

Let us note, in this 2-parameter model also fraction recovered $f(\bar{d})$ is a function of reduced density, $\bar{d}$ and not of density d. Here also $f(\bar{d}) = 1$ when $\bar{d} = \bar{d}_1$, and $f(1) = 0.5$ and $f(\infty) = 0$.

Based on the 2-parameter model equation (4.16), the effect of variation of $\bar{d}_1$, (the minimum reduced density of coal that will result in 100 per cent float) for a specific sharpness index, n is studied. The computed results for the above effect have been given in Appendix 3. The same results have been shown graphically in Figure 4.3. From Figure 4.3 it is evident that higher the $\bar{d}_1$ value, higher is the separation or cleaning of coal.

Now we come to the general case of a 4-parameter model, taking into account $f_u < 1$, $f_1 > 0$ and $d_1 > 0$. For this purpose we make a transformation as follows:

$$\frac{f - f_1}{f_u - f_1} = \bar{f} \tag{4.17}$$

Here also $\bar{f} = 1$ when $f = f_u$ and $\bar{f} = 0$ when $f = f_1$.

Equation (4.13) becomes

$$\frac{f - f_1}{f_u - f_1} = \frac{1}{1+K(d-d_1)^n} \tag{4.18}$$

At the split point,

$$\frac{0.5 - f_1}{f_u - f_1} = \frac{1}{1+K(d_{0.5} - d_1)^n} \tag{4.19}$$

or

$$K = [(\frac{f_u - f_1}{0.5 - f_1}) - 1] \frac{1}{(d_{0.5} - d_1)^n} \tag{4.20}$$

Therefore equation (4.13) becomes

$$\frac{f - f_l}{f_u - f_l} = \frac{1}{1+\left(\frac{f_u - 0.5}{0.5 - f_l}\right)\left(\frac{\bar{d} - \bar{d}_l}{1 - \bar{d}_l}\right)^n} \tag{4.21}$$

or

$$f = f(\bar{d}) = f_l + \frac{f_u - f_l}{1+\left(\frac{f_u - 0.5}{0.5 - f_l}\right)\left(\frac{\bar{d} - \bar{d}_l}{1 - \bar{d}_l}\right)^n} \tag{4.22}$$

Here also we note,

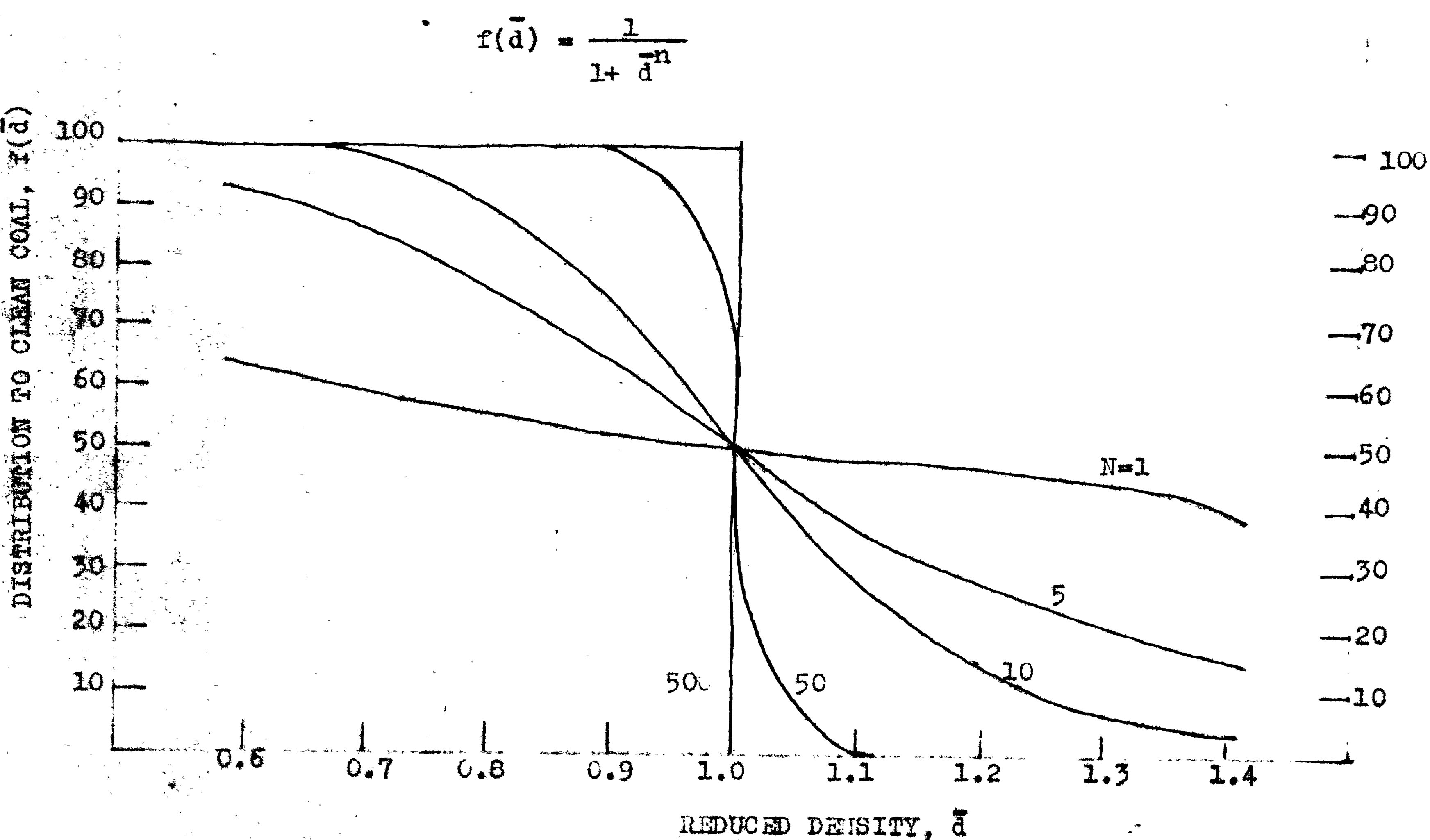
$$f(\bar{d}_l) = f_u$$

$$f(\infty) = f_l$$

$$f(1) = 0.5$$

and $f(\bar{d})$ depends on $\bar{d}$ and not on d, that is the fraction recovered is a function of reduced density is never violated in the setting up of 2- and 4-parameter models.

A representation of 4 parameter model is given in Figure 4.4.



$f(\bar{d}) = \frac{1}{1+\bar{d}^{-n}}$
DISTRIBUTION TO CLEAN COAL, $f(\bar{d})$
100
90
80
70
60
50
40
30
20
10
N=1
5
10
50
500
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
REDUCED DENSITY, $\bar{d}$

FIGURE 4.2

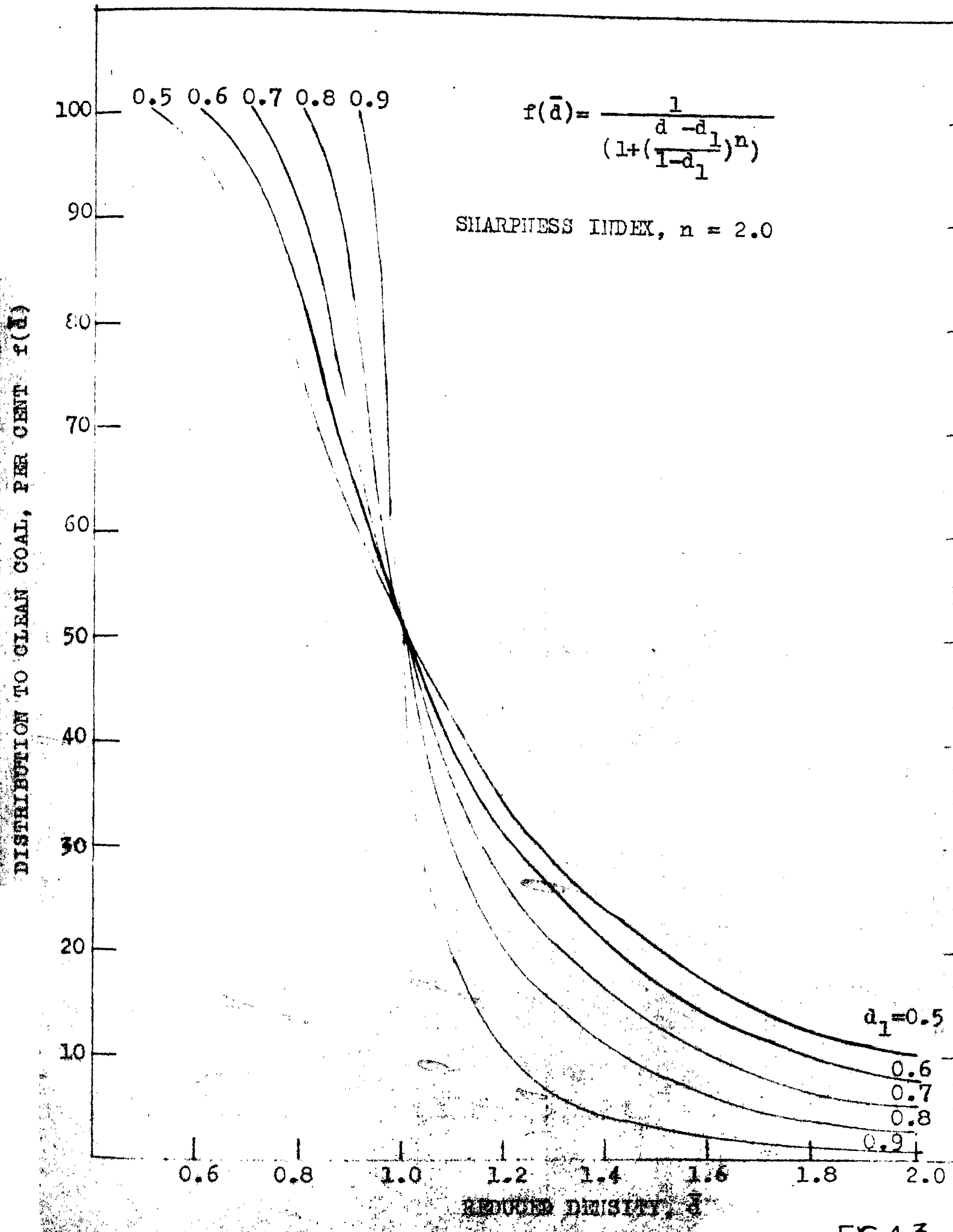

0.5 0.6 0.7 0.8 0.9
$f(\bar{d}) = \frac{1}{(1+(\frac{\bar{d}-d_1}{1-d_1})^n)}$
SHARPNESS INDEX, n = 2.0
DISTRIBUTION TO CLEAN COAL, PER CENT $f(\bar{d})$
100
90
80
70
60
50
40
30
20
10
d_1=0.5
0.6
0.7
0.8
0.9
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
REDUCED DENSITY, $\bar{d}$

FIG.4.3

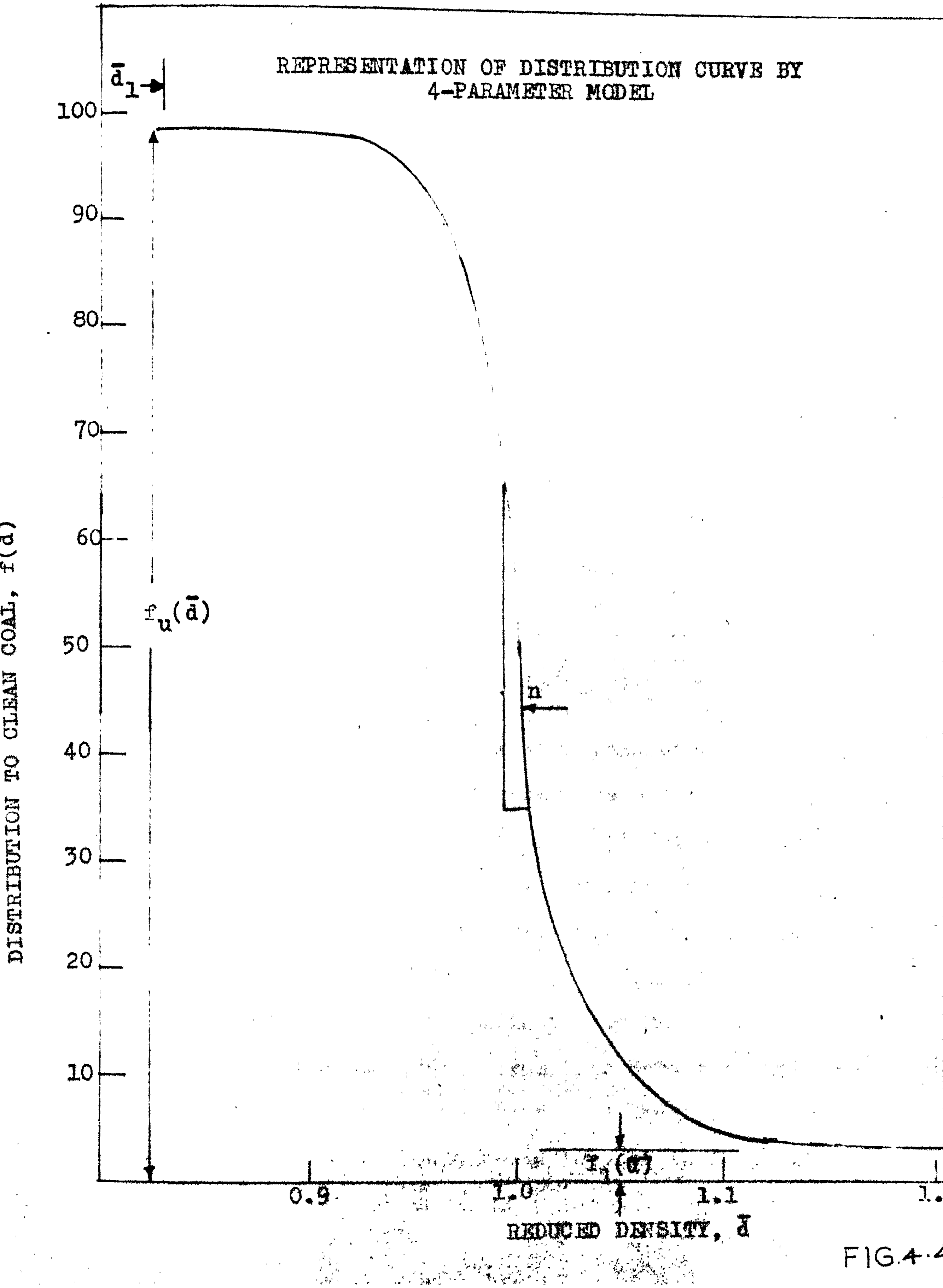

REPRESENTATION OF DISTRIBUTION CURVE BY
4-PARAMETER MODEL
$\bar{d}_1$
$f_u(\bar{d})$
n
$f_l(\bar{d})$
DISTRIBUTION TO CLEAN COAL, $f(\bar{d})$
100
90
80
70
60
50
40
30
20
10
0.9
1.0
1.1
REDUCED DENSITY, $\bar{d}$

FIG.4·4

CHAPTER 5

METHODOLOGY

In the Chapter 4, we have developed a 2-parameter and a 4-parameter model for the analysis of coal cleaning operation. The 2-parameter model is

$$f(\bar{d}) = \frac{1}{1 + \left(\frac{\bar{d} - \bar{d}_1}{1 - \bar{d}_1}\right)^n} \qquad (4.16)$$

where $\bar{d}_1$ and n are the two parameters that must be determined from the experimental data. The 4-parameter model is,

$$f(\bar{d}) = f_1 + \frac{f_u - f_1}{1 + \left(\frac{f_u - 0.5}{0.5 - f_1}\right)\left(\frac{\bar{d} - \bar{d}_1}{1 - \bar{d}_1}\right)^n} \qquad (4.22)$$

where $\bar{d}_1$, n, f_u and f_1 are the four parameters to be determined.

The parameters in our models were estimated from the rather extensive data provided by Gottfried and Jacobsen [13], where $\bar{d}$ and $f(\bar{d})$ values for various sizes ranges of coal processed in different plants are given. The breakdown of the data are given in the following Table 5.1.

Our objective is to estimate the values of 2 and 4 parameters in the two models from this data that will represent the physical situation, such that $f(\bar{d})$ calculated using 2- or 4-parameter model should be as 'close' to $f(\bar{d})$ experimental (which is available in data) as possible.

TABLE 5.1: GENERALIZED DISTRIBUTION DATA [13]

S.No.	Equipment	Feed sizes, in.
1.	Dense medium vessel	6x4, 4x2, 2x1, 1x1/2, 1/2x1/4 and composite feed (6x1/4).
2.	Baum jig	6x3, 3x1$\frac{5}{8}$, 1$\frac{5}{8}$x1/2, 1/2x1/4, 1/4x8 mesh, 8x14 mesh, 14x68 mesh and composite feed (6 x 48 mesh).
3.	Dense medium cyclone	3/4x1/2, 1/2x3/8, 3/8x1/4, 1/4x8mesh, 8x14 mesh, 14x28 mesh and composite feed (3/4 x 28 mesh).
4.	Hydrocyclone	1/4x4mesh, 4x8mesh, 8x4mesh, 14x28 mesh, 28x48 mesh, 48x100 mesh, 100x200 mesh and composite feed (1/4 x 200 mesh).
5.	Concentrating table	3/8x1/4, 1/4x8 mesh, 8x14 mesh, 14x28 mesh, 28x48 mesh, 48x100 mesh, 100x200 mesh and composite feed (3/8 x 200 mesh).

Following the standard procedure for parameter estimation, the objective function can be written as

$$\sum_{i=1}^{n} (f_i(\bar{d}) - f_i(\bar{d}))^2 \qquad (5.1)$$

where i = 1,2,...n, and n is the number of experimental points considered. By minimizing the above objective function, we can produce the best estimate of 2 and 4 parameters in our models. The optimization techniques used for the above purpose were

1. Box complex technique [14,15]
2. Rosenbrock hill climb method [16,17]

The two methods are discussed, with flow chart and computer program, in Appendix 1 and Appendix 2 respectively.

5.1 Selection of 4-Parameter Model

For a particular coal cleaning vessel and a given size range of coal, both 2-parameter and 4-parameter models were tried. Almost all the trials indicated that the 4-parameter model reproduces the experimental results better than the 2-parameter model. Example of one such case is given in Table (5.2). So we selected the 4-parameter model for future analysis in this study.

5.2 Selection of Best Weighted Objective Function

The $f(\bar{d})$ data which ranges from 0.0 per cent to 100.0 per cent may not be very accurate at both extreme ends. To

compensate for this, the weighted objective functions were also tried, namely

$$\sum_{i=1}^{n} W\,(f_i(\bar{d}) - f_i(\bar{d}))^2 \qquad (5.2)$$

where the W is the weight given as

$$W = (50 - \text{Abs}\,(f_i(\bar{d}) - 50))/50$$

This weight will give a value of 1 when $f_i(\bar{d}) = 50$, and a value of zero at $f_i(\bar{d}) = 0$ and 100. That is, more weightage is given to the central portion of the data than the end points in the two tails. In some trials the extreme points were heavily weighted in order to force fit the curve in the tail region.

The weightage for the extreme points can be given by just replacing W by (1-W) in the objective function. For uniform weightage throughout the experimental data points, value of unity is given for W.

The extent to which the model reproduces the experimental data was determined interms of absolute error per data point and normalized error per data point:

$$\text{Absolute error} = \frac{\sum_{i=1}^{n} (f_i(\bar{d}) - f_i(\bar{d}))^2}{n}$$

$$\text{Normalized error} = \frac{\sum_{i=1}^{n} \frac{(f_i(\bar{d}) - f_i(\bar{d}))^2}{f_i(\bar{d})}}{n}$$

Many trials conducted in this manner showed that the objective function weighted uniformly gives the best reproduction of experimental results. So, we selected uniform weight, that is no weight is imposed on any part of the distribution curve. Example of uniform weight, weighted at mid portion and weighted at extreme points were given in Table 5.3.

It was also determined that the absolute error criterion in general gave better results than normalized error criterion. Example to the effect is given in Table 5.4.

For every data set, estimation of parameter was carried out for 0-100 per cent, 1-99 per cent and 2-98 per cent range of $f(\bar{d})$. As we expected, the 2-98 per cent range gave best reproduction. Example of this is given in Table 5.5.

5.3 Selection of Rosenbrock Hill Climb Method

For the 4-parameter model, and uniformly weighted objective function, both Box Complex technique and Rosenbrock Hill Climbing Method were tried for each data set. It was found that in some cases both methods give identical results. In other cases however, for box-complex method to be successful, (i) the reflection coefficient, the expansion coefficient and the contraction co-efficient had to be properly selected, (ii) moreover the success of the method was quite sensitive to the choice of the initial guess points, and (iii) also box-complex technique took more computer time than

the Rosenbrock hill climb procedure. Consequently, Rosenbrock hill climb procedure was used for estimating the 4-parameters in our model. An example to this effect is given in Table 5.6.

The maximum and minimum values of the 4-parameters should be provided for the estimation of the 4-parameters. From the studies on nature of distribution curves for various sharpness index, n as in Appendix 3, n around 50 is found to give experimental conditions. So the range given for n is 0-100. Since $\bar{d}_1$ can be any value between 0 to minimum reduced density experimentally found, the range given for $\bar{d}_1$ is 0- $\bar{d}_i$ when $\bar{d}_i$ is the minimum reduced density in data for which the 4 parameters should be estimated. Also to give a wider choice, $f_u(\bar{d})$ is given a range between 50 - 100 per cent and $f_1(\bar{d})$ between 0 - 50 per cent.

Since the optimization technique used in the estimation of 4 parameters is heuristic sometimes we got too high n values and too low $\bar{d}_1$ values. Such unrealistic values are discarded and reasonable values of the 4 parameters were selected and were listed out in Appendix 4.

TABLE 5.2

[illegible] HILLCLIMB PROCEDURE

[illegible] DENSE MEDIUM CYCLONE - COMP.FEED(3/4" X 28-MESH)

[illegible] X(1)=0.8, X(2)=10., X(3)=99., X(4)=1.0

[illegible] E(1)=0.005,E(2)=0.05,E(3)=0.01,E(4)=0.005

```
***************************************
        2PARAMETER MODEL
***************************************

[illegible] EVALUTIONS =        134
[illegible] THIS STAGE
X( 1 ) =  .823117E+00
X( 2 ) =  .11[illegible]E+02
```

DX(I)	FD(I)	FCAL	DIFF
0.8232	100.0000	100.0000	0.0000
0.8684	99.8000	100.0000	0.2000
0.9130	99.2000	99.9576	0.7576
0.9377	98.6000	99.3171	0.7171
0.9474	98.0000	98.2838	0.2838
0.9559	97.0000	96.4349	-0.5651
0.9609	96.0000	94.6093	-1.3907
0.9658	95.0000	92.1600	-2.8400
1.0000	50.0000	50.0000	0.0000
1.0230	19.7000	19.7495	0.0495
1.0285	15.0000	15.2730	0.2730
1.0352	11.2000	11.0906	-0.1094
1.0411	9.4000	8.3461	-1.0539
1.0515	7.5000	5.0654	-2.4346
1.0706	5.0000	2.0797	-2.9203
1.0800	3.9000	1.3661	-2.5339
1.1007	2.8000	0.5662	-2.2338
1.1471	1.7000	0.0966	-1.6034
1.2176	1.0000	0.0101	-0.9899

ABS.ERROR IS= 0.22181351E+01   NOR.ERROR IS = 0.17835928E+00

```
***************************************
        4PARAMETER MODEL
***************************************

NUMBER OF FUNCTION EVALUTIONS =        290
VALUES OF X AT THIS STAGE
X( 1 ) =  0.822082E+00
X( 2 ) =  0.125501E+02
X( 3 ) =  0.998402E+02
X( 4 ) =  0.262692E+01
```

DX(I)	FD(I)	FCAL	DIFF
0.8232	100.0000	99.8402	-0.1598
0.8684	99.8000	99.8402	0.0402
0.9130	99.2000	99.8178	0.6178
0.9377	98.6000	99.3848	0.7848
0.9474	98.0000	98.5986	0.5986
0.9559	97.0000	97.0816	0.0816
0.9609	96.0000	95.5013	-0.4987
0.9658	95.0000	93.2938	-1.7062
1.0000	50.0000	50.0000	-0.0000
1.0230	19.7000	19.2784	-0.4216
1.0285	15.0000	15.1061	0.1061
1.0352	11.2000	11.3546	0.1546
1.0411	9.4000	8.9879	-0.4121
1.0515	7.5000	6.2862	-1.2138
1.0706	5.0000	4.0010	-0.9990
1.0800	3.9000	3.4938	-0.4062
1.1007	2.8000	2.9577	0.1577
1.1471	1.7000	2.6749	0.9749
1.2176	1.0000	2.6310	1.6310

ABS.ERROR IS= 0.58967842E+00   NOR.ERROR IS = 0.16170142E+0[illegible]

TABLE 5.3

[illegible] : HYDROCYCLONE -COMP.FEED (1/4" X 200 MESH)

[illegible] : X(1)=0.66,X(2)=2.3,X(3)=92.0,X(4)=3.0

[illegible] : E(1)=0.005,E(2)=0.05,E(3)=0.01,E(4)=0.005

: ROSENBROCK HILLCLIMB METHOD

****
WEIGHTED - UNIFORMLY
****

X(1) = 0.655795E+00 X(2) = 0.242335E+01
X(3) = 0.925[illegible]E+02 X(4) = 0.357[illegible]E+01

DX(I)	FD(I)	FCAL	DIFF
0.6850	93.6000	92.4199	-1.1801
0.7280	90.3000	91.1371	0.8371
0.7720	86.3000	87.7123	1.4123
0.8200	81.2000	81.5466	0.3466
0.8610	76.1000	74.7869	-1.3131
0.8935	71.0000	68.9006	-2.0994
1.0000	50.0000	50.0000	0.0000
1.0251	45.0000	46.1013	1.1013
1.0340	43.3000	44.7877	1.4877
1.0590	39.6000	41.2954	1.6954
1.1000	35.3000	36.1926	0.8926
1.1670	30.4000	29.3942	-1.0058
1.2120	27.3000	25.7493	-1.5507
1.2550	24.4000	22.8353	-1.5647
1.2990	21.6000	20.3324	-1.2676
1.3430	18.9000	18.2301	-0.6699
1.3880	16.3000	16.4204	0.1204
1.4330	14.2000	14.8933	0.6933
1.4760	12.6000	13.6512	1.0512
1.5200	11.6000	12.5621	0.9621

ABS.ERROR TS= 0.13941436E+01 NOR.ERROR TS = 0.17682886E-02

****
WEIGHTED - EXTREMELY
****

X(1) = 0.656564E+00 X(2) = 0.238867E+01
X(3) = 0.930975E+02 X(4) = 0.234536E+01

DX(I)	FD(I)	FCAL	DIFF
0.6850	93.6000	92.8843	-0.7157
0.7280	90.3000	91.2088	0.9088
0.7720	86.3000	87.4084	1.1084
0.8200	81.2000	81.0233	-0.1767
0.8610	76.1000	74.2600	-1.8400
0.8935	71.0000	68.4617	-2.5383
1.0000	50.0000	50.0000	0.0000
1.0251	45.0000	46.1798	1.1798
1.0340	43.3000	44.8897	1.5897
1.0590	39.6000	41.4506	1.8506
1.1000	35.3000	36.3968	1.0968
1.1670	30.4000	29.5952	-0.8048
1.2120	27.3000	25.9082	-1.3918
1.2550	24.4000	22.9361	-1.4639
1.2990	21.6000	20.3635	-1.2365
1.3430	18.9000	18.1868	-0.7132
1.3880	16.3000	16.3002	0.0002
1.4330	14.2000	14.6979	0.4979
1.4760	12.6000	13.3872	0.7872
1.5200	11.6000	12.2316	0.6316

****
[illegible] - [illegible]
****

[illegible] EVALUATIONS = 460

Y(1) = 0.590[illegible]E+00   Y(2) = 0.230268E+01
Y(3) = 0.[illegible]E+02   Y(4) = 0.165101[illegible]E+01

X(I)	FD(I)	ECAL	DIFF
[illegible]	93.6000	91.2505	-2.3495
[illegible]	89.3000	89.7893	0.4893
[illegible]	86.3000	88.0539	1.7539
[illegible]	81.2000	82.2036	1.0036
[illegible]	76.1000	75.4093	-0.6907
[illegible]	71.0000	69.3843	-1.6157
[illegible]	59.0000	59.0000	0.0000
[illegible]	45.0000	46.0409	1.0409
[illegible]	43.3000	44.7117	1.4117
[illegible]	39.6000	41.1898	1.5898
[illegible]	35.3000	36.0770	0.7770
[illegible]	30.4000	29.3291	-1.0709
[illegible]	27.3000	25.7412	-1.5588
[illegible]	24.4000	22.8869	-1.5131
[illegible]	21.6000	20.4447	-1.1553
[illegible]	18.9000	18.3994	-0.5006
[illegible]	16.3000	16.6425	0.3425
[illegible]	14.2000	15.1620	0.9620
[illegible]	12.6000	13.9590	1.3590
[illegible]	11.6000	12.9044	1.3044

ABS.ERROR TS= 0.15590492E+01   NOR.ERROR TS = 0.23368678E-02

STUDENT ... ABSOLUTE ERROR CRITERION

TABLE - 5.4.

... MEDIUM CYCLONE - 3/4 '' X 1/2 ''

X(1)= [illegible]   X(2)= .10790000E+02   X(3)=99.18000   X(4)= 0.00500

D(I)	FD(I)	FCAL	DIFF
[illegible]	100.00	99.18	0.82
[illegible]	99.70	99.12	0.58
[illegible]	99.00	98.70	0.30
[illegible]	98.30	97.97	0.33
[illegible]	97.80	97.14	0.66
[illegible]	96.40	96.79	-0.39
1.[illegible]	50.00	50.00	0.00
1.[illegible]	8.90	7.96	0.94
1.[illegible]	5.40	6.68	-1.28
1.[illegible]	4.00	5.15	-1.15
1.[illegible]	3.00	3.35	-0.35
1.[illegible]	2.00	1.73	0.27
1.[illegible]	1.20	0.85	0.35
1.[illegible]	0.60	0.41	0.19
1.[illegible]	0.20	0.21	-0.01

STANDARD DEVIATION = 0.16866186E+00
ABSOLUTE ERROR = 0.39825551E+00
NORMALISED ERROR = 0.24384398E-01

X(1)= 0.8526   X(2)= .17160000E+02   X(3)=99.99000   X(4)= 0.19200

D(I)	FD(I)	FCAL	DIFF
0.9160	100.00	99.95	0.05
0.9550	99.70	99.80	-0.10
0.9550	99.00	99.04	-0.04
0.9700	98.30	98.01	0.29
0.9730	97.80	96.97	0.83
0.9740	96.40	96.53	-0.13
1.0000	50.00	50.00	0.00
1.0230	8.90	7.82	1.08
1.0250	5.40	6.52	-1.12
1.0280	4.00	4.98	-0.98
1.0330	3.00	3.20	-0.20
1.0410	2.00	1.64	0.36
1.0500	1.20	0.85	0.35
1.0600	0.60	0.47	0.13
1.0700	0.20	0.32	-0.12

STANDARD DEVIATION = 0.14643861E+00
ABSOLUTE ERROR = 0.30021973E+00
NORMALIZED ERROR = 0.42150164E-01

TABLE 6.5

[illegible] : [illegible] CYCLONE - 3/8 " X 1/4 ")
[illegible] : X(1)=1.0,X(2)=10.5,X(3)=96.5,X(4)=0.1
[illegible] : E(1)=0.005,E(2)=0.05,E(3)=0.01,E(4)=0.005
[illegible] PROCEDURE

****
> 0.0 % & UPTO 100.0 % CONSIDERED
****

[illegible] FUNCTION EVALUTIONS = 235

X(1) = [illegible]E+00 X(2) = 0.1095[illegible]E+02
X(3) = [illegible]E+02 X(4) = 0.5[illegible]E-02

DX(I)	FD(I)	FCAL	DIFF
0.9020	100.0000	98.6561	-1.3439
[illegible]	99.2000	98.6474	-0.5526
0.9500	98.7000	98.5773	-0.1227
0.9560	98.2000	98.4105	0.2105
0.9630	97.3000	97.8498	0.5498
0.9650	96.8000	97.5507	0.7507
0.9720	95.1000	95.5701	0.4701
1.0000	50.0000	50.0000	0.0000
1.0300	5.3000	6.0415	0.7415
1.0330	4.4000	4.8112	0.4112
1.0370	3.4000	3.5646	0.1646
1.0480	2.0000	1.6076	-0.3924
1.0620	1.0000	0.6247	-0.3753
1.0750	0.5000	0.2783	-0.2217

ABS.ERROR IS= 0.31051035E+00 NOR.ERROR IS = 0.29065136E-01

****
> 1.0 % & < 99.0 % CONSIDERED
****

NUMBER OF FUNCTION EVALUTIONS = 1578

X(1) = 0.921265E+00 X(2) = 0.193391E+02
X(3) = 0.985306E+02 X(4) = 0.934575E+00

DX(I)	FD(I)	FCAL	DIFF
0.9500	98.7000	98.3618	-0.3382
0.9560	98.2000	98.1242	-0.0758
0.9630	97.3000	97.4557	0.1557
0.9650	96.8000	97.1260	0.3260
0.9720	95.1000	95.0822	-0.0178
1.0000	50.0000	50.0000	0.0000
1.0300	5.3000	5.5226	0.2226
1.0330	4.4000	4.4336	0.0336
1.0370	3.4000	3.3691	-0.0309
1.0480	2.0000	1.8191	-0.1809

ABS.ERROR IS= 0.33533126E-01 NOR.ERROR IS = 0.10113375E-02

****
> 2.0 % & < 98.0 % CONSIDERED
****

NUMBER OF FUNCTION EVALUTIONS = 237

X(1) = 0.899567E+00 X(2) = 0.111061E+02
X(3) = 0.976599E+02 X(4) = 0.221371E+00

DX(I)	FD(I)	FCAL	DIFF
0.9630	97.3000	97.0963	-0.2037
0.9650	96.8000	96.8662	0.0662
0.9720	95.1000	95.2470	0.1470
1.0000	50.0000	50.0000	0.0000
1.0300	5.3000	5.5028	0.2028
1.0330	4.4000	4.3743	-0.0257
1.0370	3.4000	3.2490	-0.1510

[illegible] 5.6

```
          . [illegible] COLUMN ([illegible]) =3/4'' X 1/2''
[illegible] Y(1)=0.0,Y(2)=5.,Y(3)=95.0,Y(4)=2.0
          . F(1)=0.005,F(2)=0.005,F(3)=0.01,F(4)=0.05

*******************************************
   [illegible] HILLCLIMB PROCEDURE
*******************************************

[illegible] EVALUATIONS =    20[illegible]
Y( [illegible] ) = [illegible].8504[illegible]E+00       Y( 2 ) = 0.1784[illegible]E+02
Y( [illegible] ) = [illegible].[illegible]E+02           Y( 4 ) = 0.3579[illegible]E+00
```

D(I)	FD(I)	FCAL	DIFF
[illegible]	100.0000	99.9612	-0.0388
[illegible]	99.7000	99.8269	0.1269
[illegible]	99.0000	99.1422	0.1422
[illegible]	98.3000	98.1843	-0.1157
[illegible]	97.8000	97.2050	-0.5950
[illegible]	96.4000	96.7834	0.3834
[illegible]	50.0000	50.0000	0.0000
[illegible]	8.9000	7.5012	-1.3959
[illegible]	5.4000	6.2542	0.8542
[illegible]	4.0000	4.7764	0.7764
[illegible]	3.0000	3.0969	0.0969
[illegible]	2.0000	1.6508	-0.3492
[illegible]	1.2000	0.9295	-0.2705
[illegible]	0.6000	0.5975	-0.0025
[illegible]	0.2000	0.4624	0.2624

[illegible] IS= 0.27376307E+00   NOR.ERROR IS = 0.12605103E+00

```
*******************************************
     BOX COMPLEX TECHNIQUE
*******************************************

Y(1)= 0.27178500E+00
Y(2)= 0.87925731E+02
Y(3)= .[illegible]E+02
Y(4)= 0.[illegible]E+00

[illegible] EVALUATION   200
```

D(I)	FD(I)	FCAL	DIFF
0.9460	100.00	99.89	-0.11
0.9550	99.70	99.64	-0.06
0.9650	99.00	98.71	-0.29
0.9700	98.30	97.60	-0.70
0.9730	97.80	96.53	-1.27
0.9740	96.40	96.09	-0.31
1.0000	50.00	50.00	0.00
1.0230	8.90	5.85	-2.05
1.0250	5.40	5.68	0.28
1.0230	4.00	4.34	0.34
1.0330	3.00	2.89	-0.11
1.0410	2.00	1.75	-0.25
1.0500	1.20	1.26	0.06
1.0600	0.60	1.08	0.42
1.0700	0.20	1.02	0.[illegible]2

CHAPTER 6

RESULTS

The theoretical model developed in Chapter 4 for coal cleaning processes was tested on computer using the rather extensive data provided by Gottfried and Jacobsen [13]. Based on the trials reported in Chapter 5 on methodology, it was decided to use Rosenbrock hill climb method for the estimation of 4 parameters in the model. From the estimated values of the parameters for a given size of raw coal feed in a coal cleaning equipment, the absolute error, standard deviation and the range of washed coal percentage, $f(\bar{d})$ covered were computed. The mathematical representation of the distribution curves suggested recently by Gottfried [12], by Weibull function were also used to find similar absolute error, standard deviation and the range of $f(\bar{d})$ covered for the same sets of experimental data. The comparative results of both Gottfried equation and our model for all five types of coal cleaning equipment and various sizes of coal feed are given in Appendix 5. Figures 6.1 - 6.5 show the comparison between the two models.

It is evident from the above results that our model covers a wider range of washed coal percentage $f(\bar{d})$ than it is possible by Gottfried equation. Also the absolute error and standard deviation are far less in our model. Invariably even within a given range of applicability our model is found

to be better in reproducing the distribution curves. It is to be noted that in our model, unlike Gottfried equation, the distribution curves do not break sharply at the extreme points, and the disparity between the predicted and experimental points in the distributions is never very large in our model. Moreover in our model the 4 parameters have physical significance, namely,

$f_u(\bar{d})$ - Maximum washed coal percentage that can be expected for a particular size range of raw coal feed in a given coal cleaning equipment,

$f_l(\bar{d})$ - the minimum coal percentage that will be in the sink

n - the sharpness of separation

and $\bar{d}_l$ - the minimum reduced density of the coal that will complctely float in the process.

The four parameters of our coal cleaning process model have been estimated for all five types of coal cleaning equipment and all sizes of coal in the given data. The values of these parameters, the absolute error and standard deviation are given in Appendix 4. Figures 6.6 - 6.11 show the effect of feed size on sharpness index, in different coal cleaning equipments.

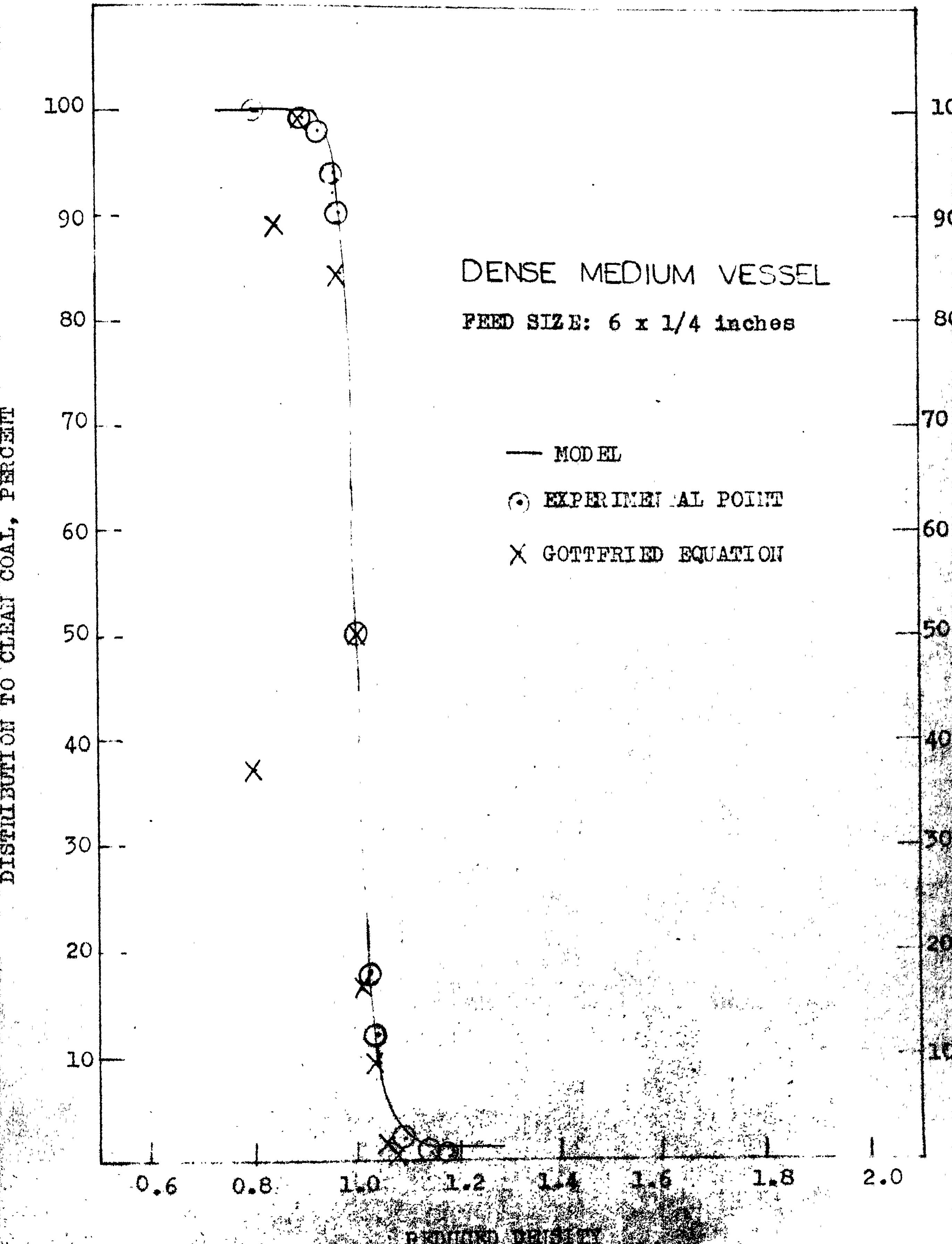
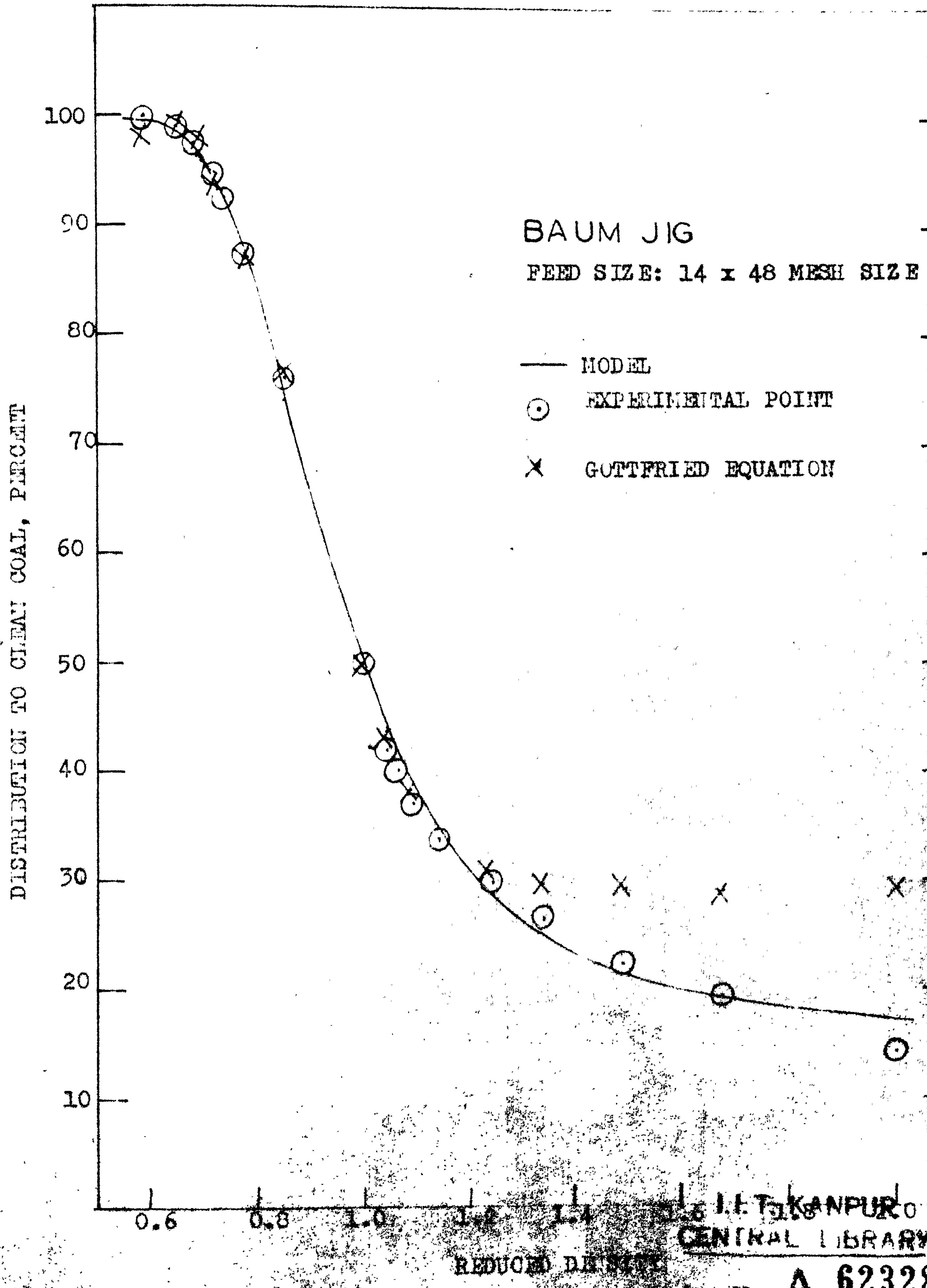
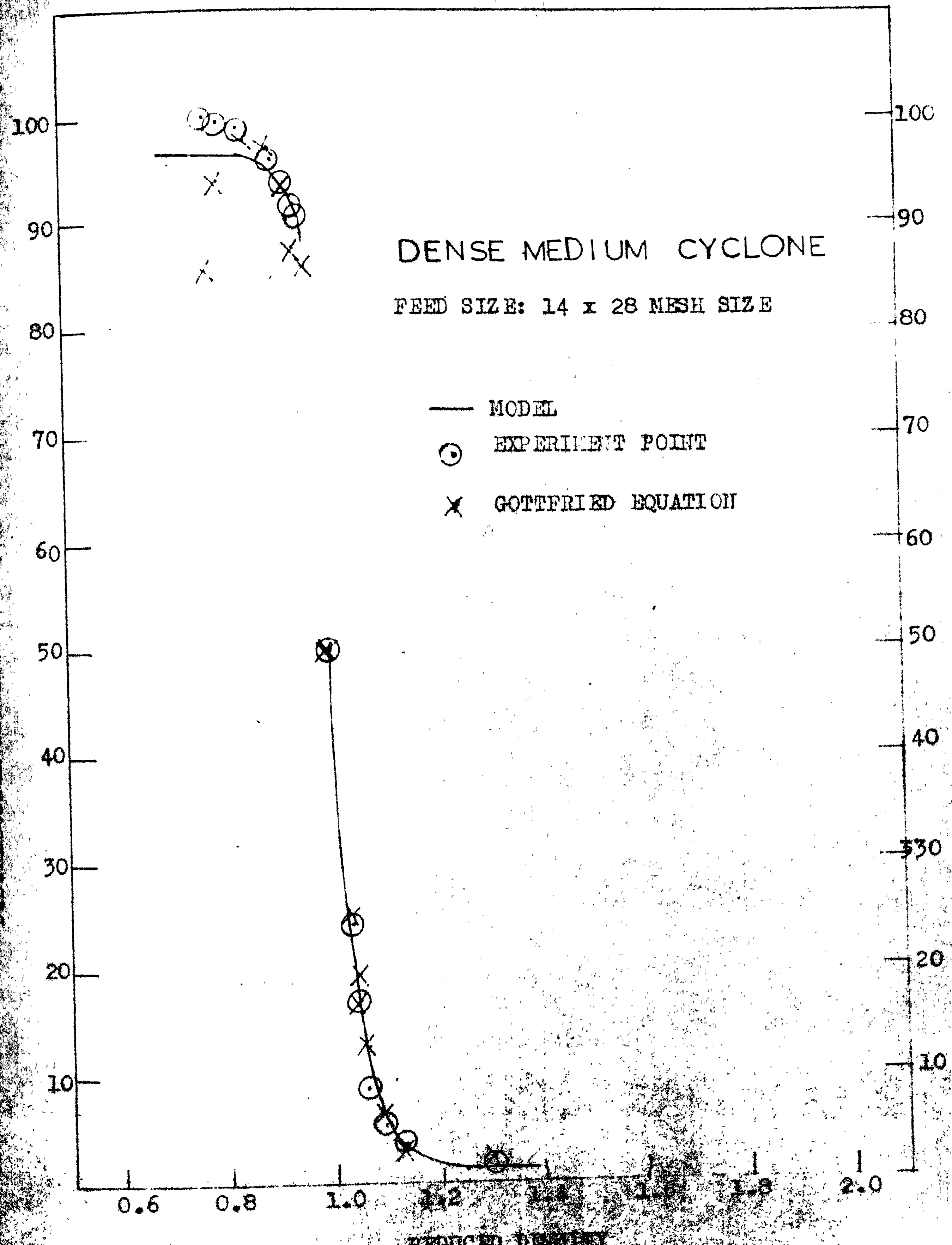
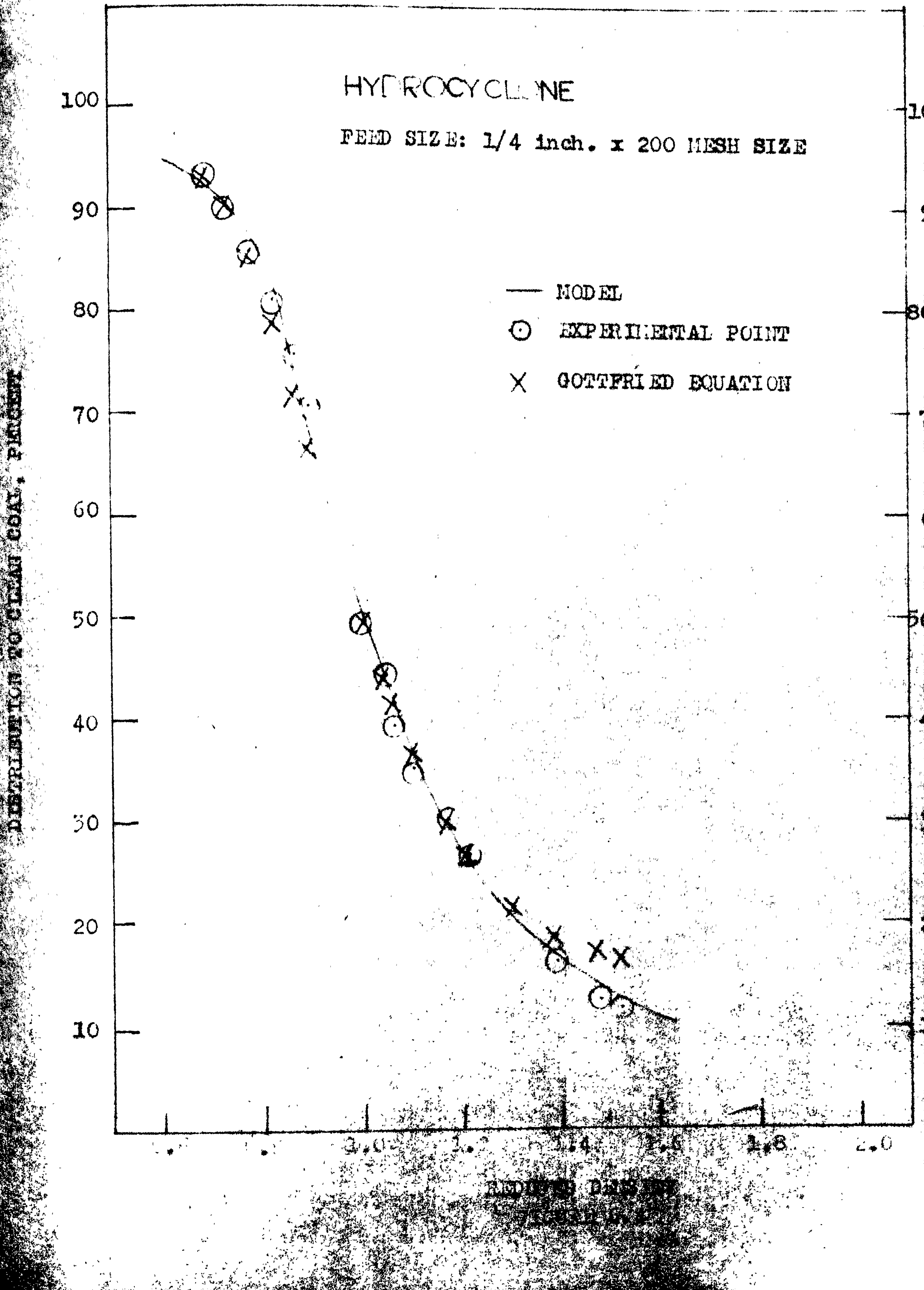

DENSE MEDIUM VESSEL
FEED SIZE: 6 x 1/4 inches
MODEL
EXPERIMENTAL POINT
GOTTFRIED EQUATION
DISTRIBUTION TO CLEAN COAL, PERCENT
100
90
80
70
60
50
40
30
20
10
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
REDUCED DENSITY

FIGURE 6.1

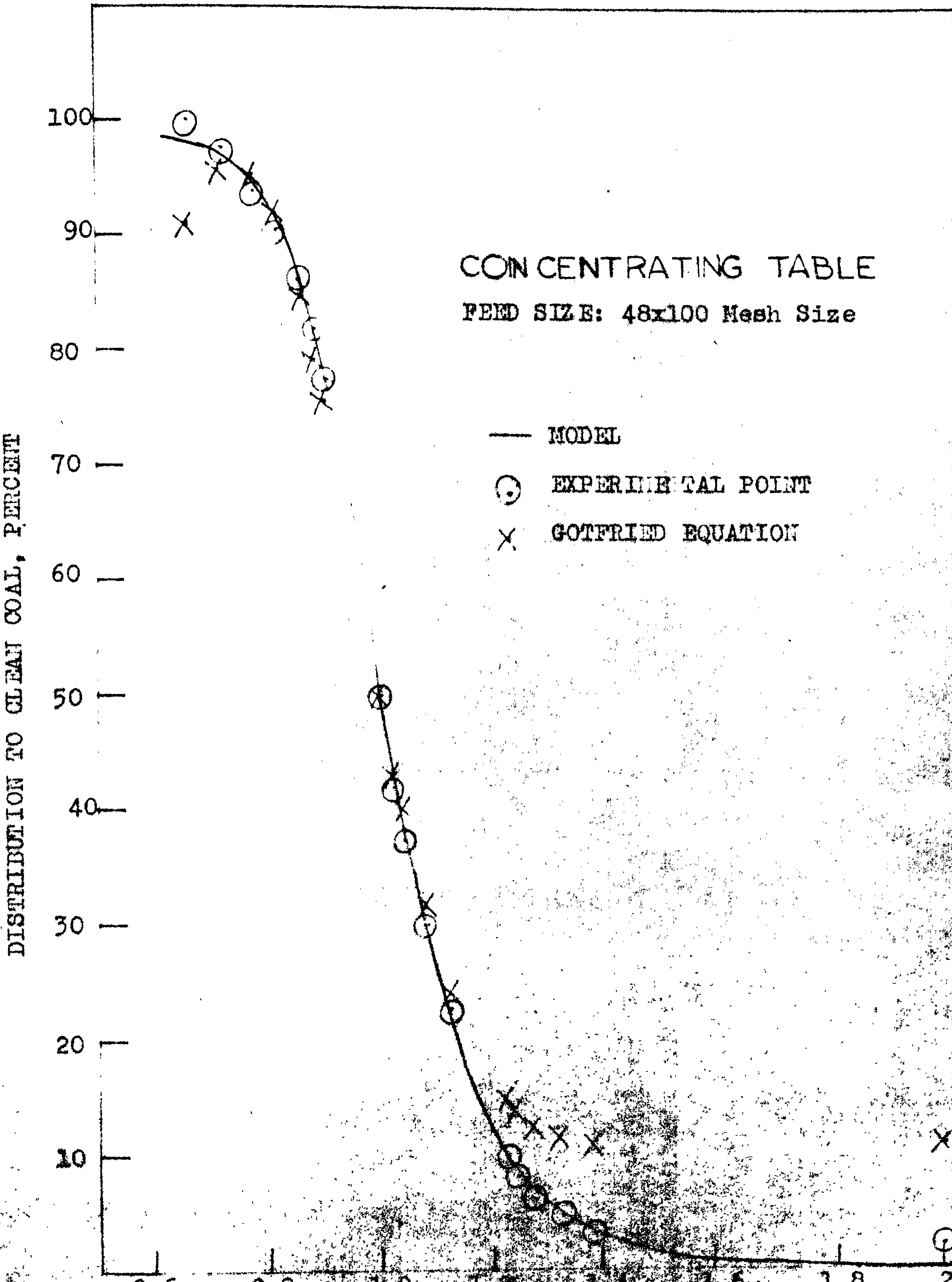

BAUM JIG
FEED SIZE: 14 x 48 MESH SIZE
MODEL
EXPERIMENTAL POINT
GOTTFRIED EQUATION
DISTRIBUTION TO CLEAN COAL, PERCENT
100
90
80
70
60
50
40
30
20
10
0.6
0.8
1.0
1.2
1.4
REDUCED DE

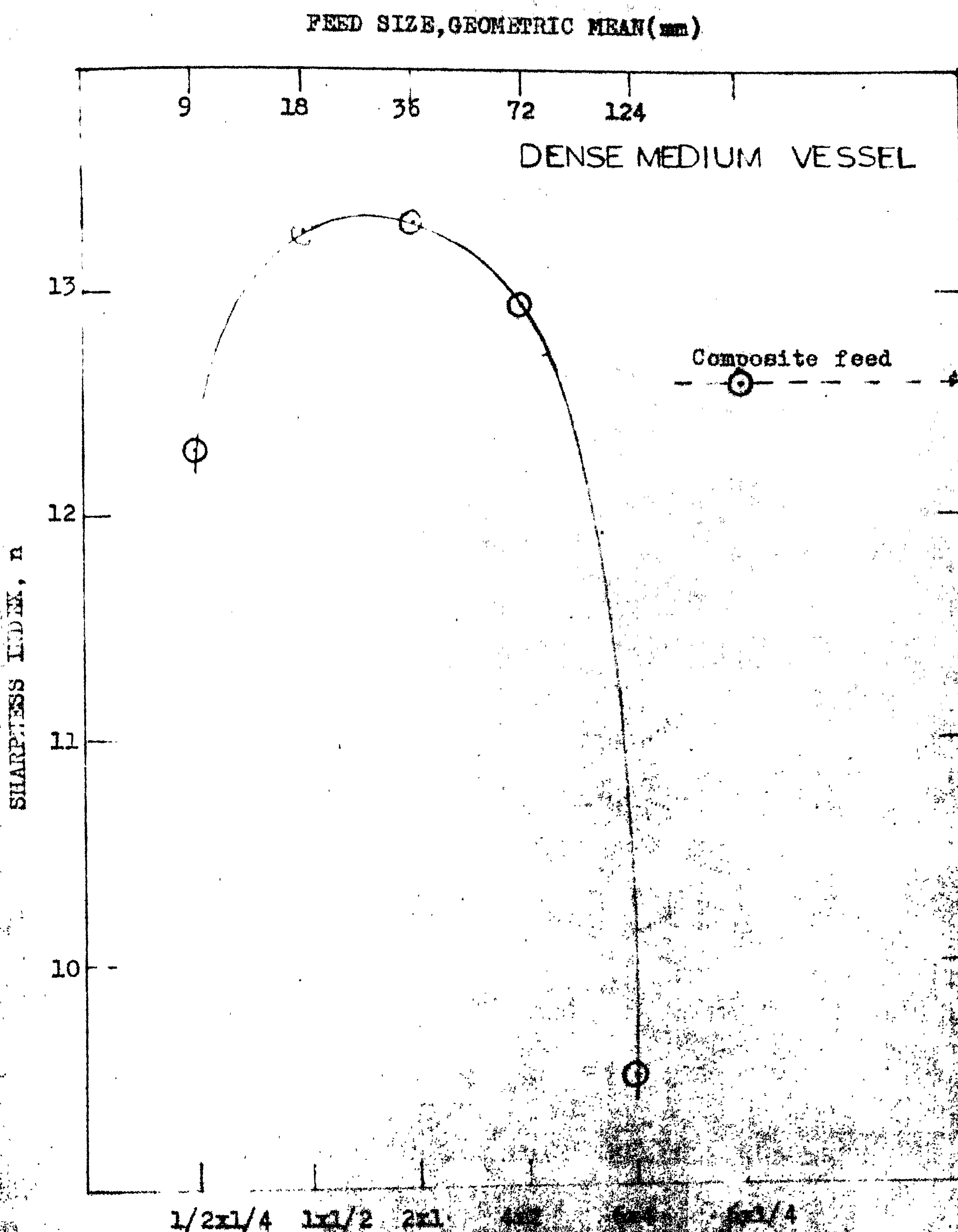

DENSE MEDIUM CYCLONE
FEED SIZE: 14 x 28 MESH SIZE
MODEL
EXPERIMENT POINT
GOTTFRIED EQUATION
100
90
80
70
60
50
40
30
20
10
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
REDUCED DENSITY

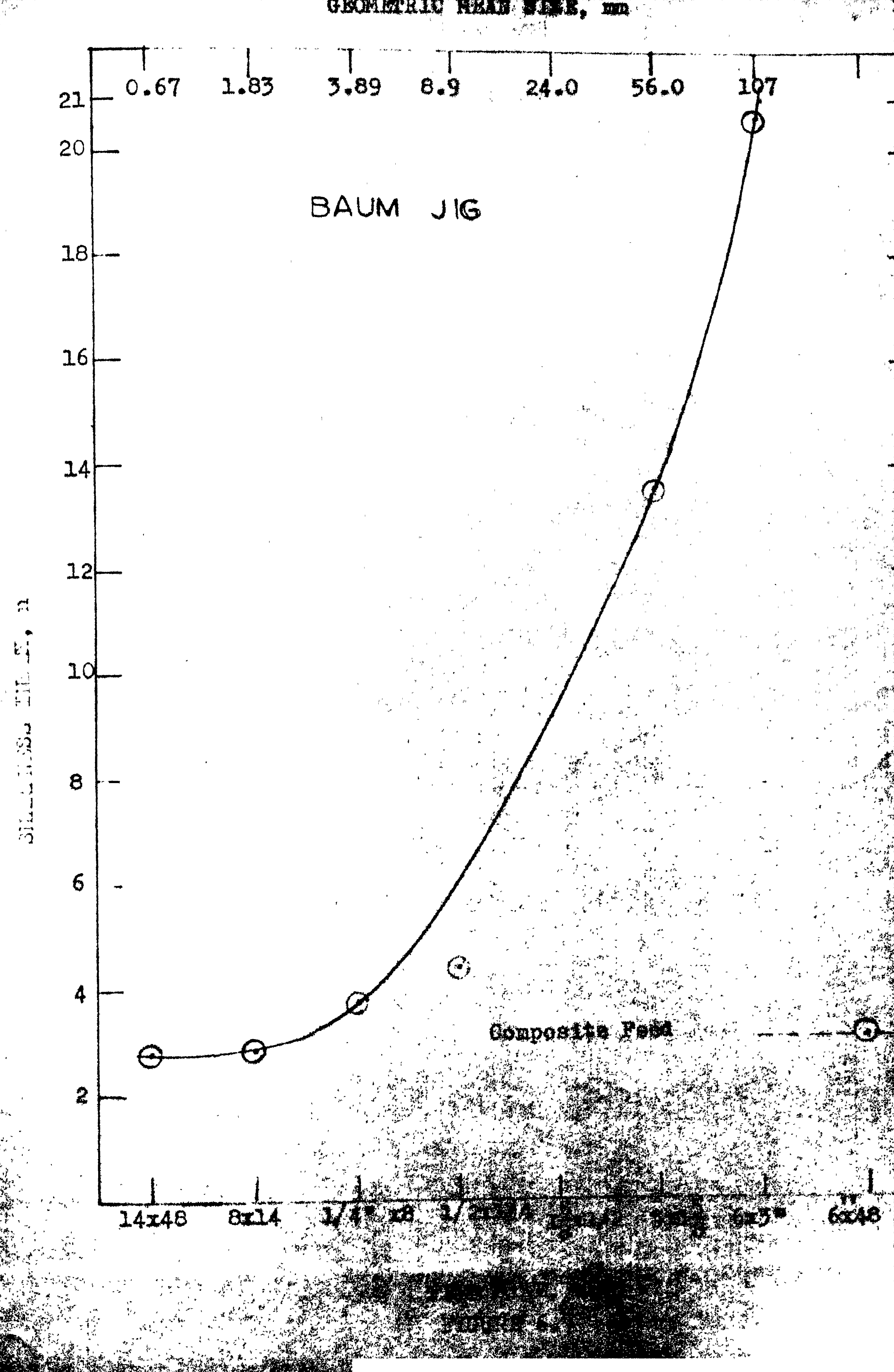
FIGURE



HYDROCYCLONE
FEED SIZE: 1/4 inch. x 200 MESH SIZE
MODEL
EXPERIMENTAL POINT
GOTTFRIED EQUATION
100
90
80
70
60
50
40
30
20
10
1.4
1.8
2.0
DISTRIBUTION TO CLEAN COAL, PERCENT


CONCENTRATING TABLE
FEED SIZE: 48x100 Mesh Size
MODEL
EXPERIMENTAL POINT
GOTFRIED EQUATION
DISTRIBUTION TO CLEAN COAL, PERCENT
100
90
80
70
60
50
40
30
20
10
0.6
0.8
1.0
1.8


FEED SIZE,GEOMETRIC MEAN(mm)
9
18
36
72
124
DENSE MEDIUM VESSEL
Composite feed
13
12
11
10
SHARPNESS INDEX, n
1/2x1/4
1x1/2
2x1


GEOMETRIC MEAN SIZE, mm
0.67
1.83
3.89
8.9
24.0
56.0
107
BAUM JIG
21
20
18
16
14
12
10
8
6
4
2
Composite Feed
14x48
8x14
6x48

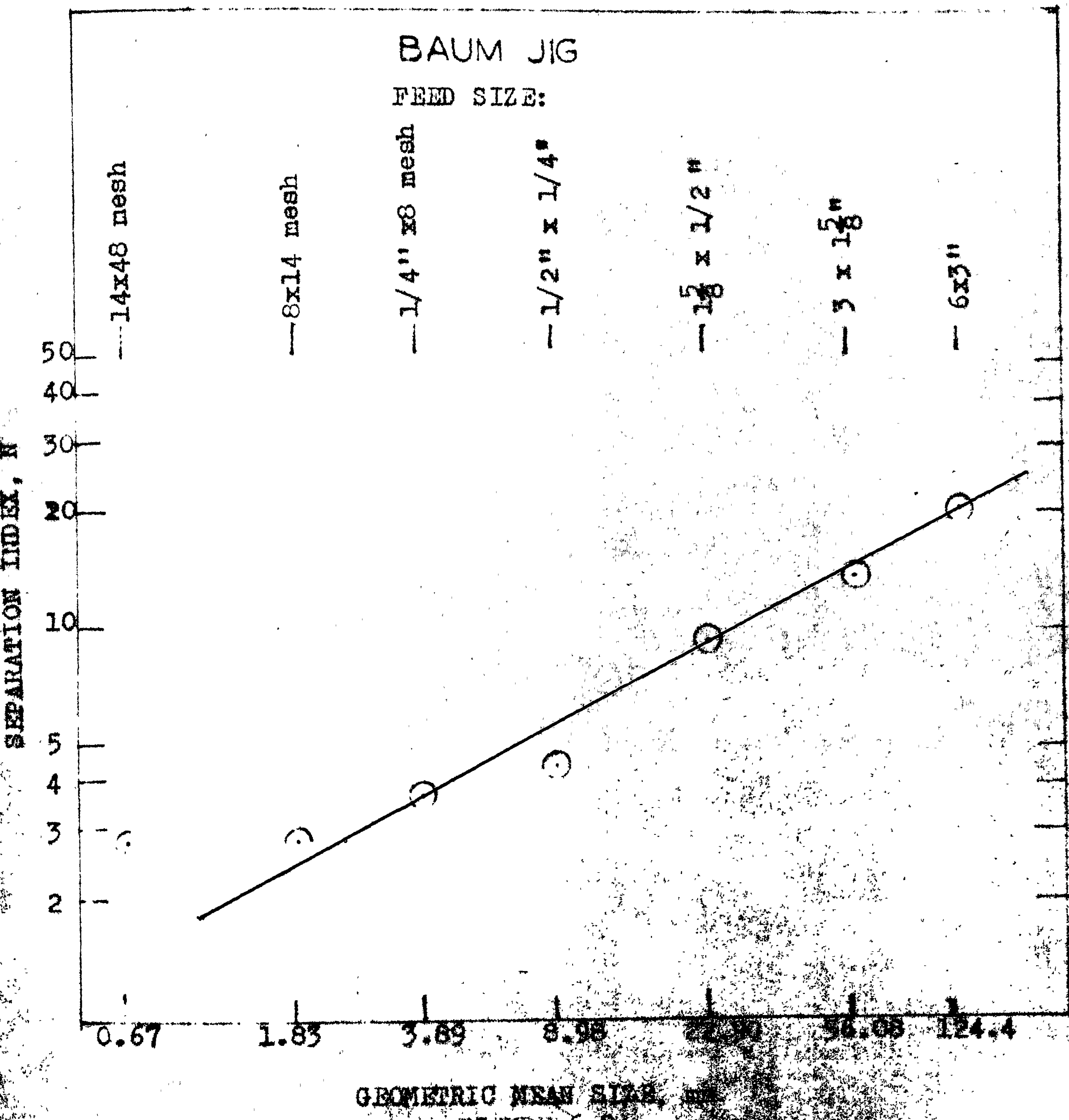

BAUM JIG
FEED SIZE:
14x48 mesh
8x14 mesh
1/4" x8 mesh
1/2" x 1/4"
1 5/8 x 1/2"
3 x 1 5/8"
6x3"
SEPARATION INDEX, N
50
40
30
20
10
5
4
3
2
0.67
1.83
3.89
8.98
22.90
56.08
124.4
GEOMETRIC MEAN SIZE, mm

FIGURE 6.8

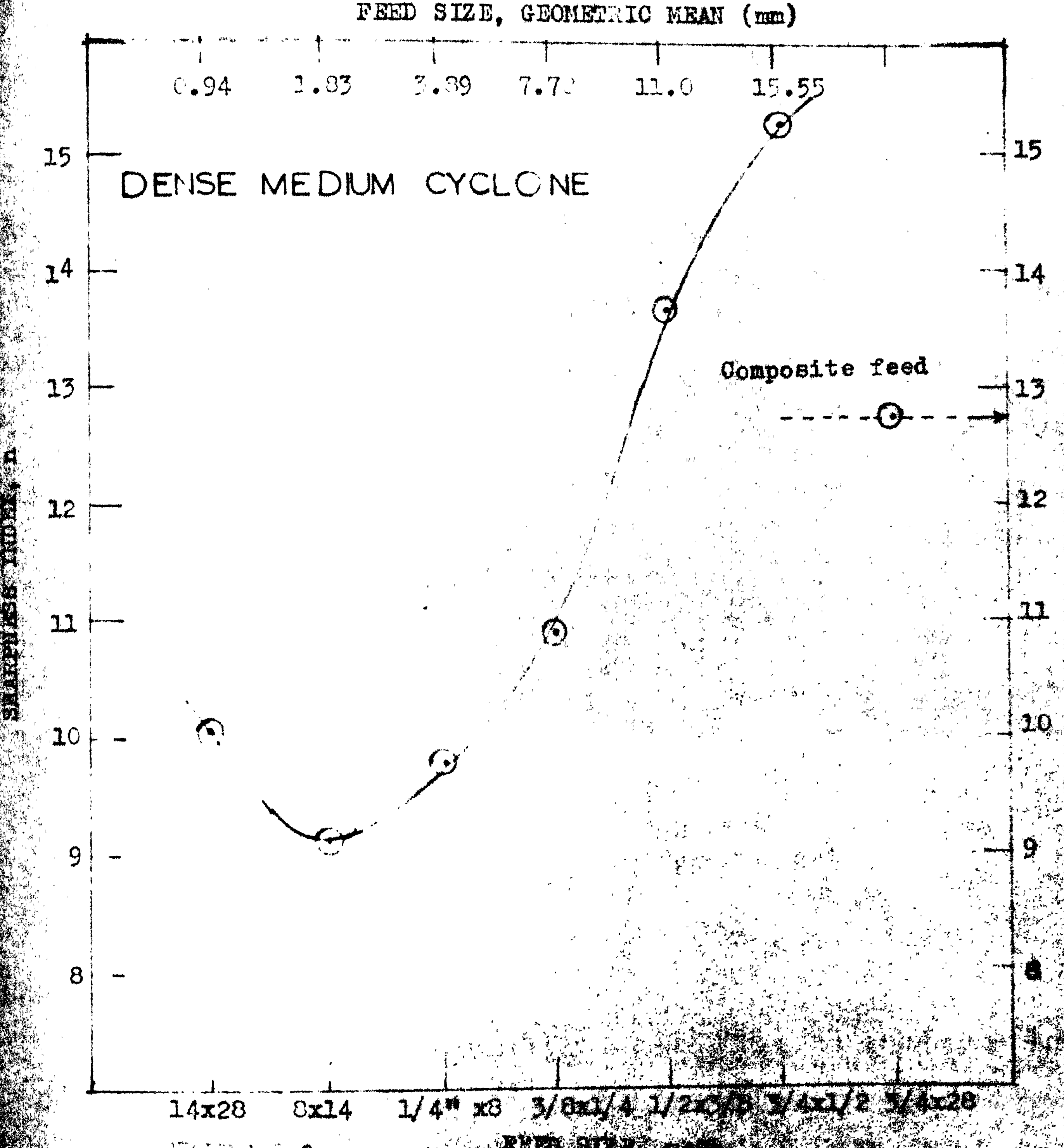

FEED SIZE, GEOMETRIC MEAN (mm)
0.94
1.83
3.89
7.72
11.0
15.55
DENSE MEDIUM CYCLONE
Composite feed
SHARPNESS INDEX, n
15
14
13
12
11
10
9
8
14x28
8x14
1/4" x8
3/8x1/4
1/2x3/8
3/4x1/2
3/4x28
FEED SIZE, mesh

FIGURE 6.9

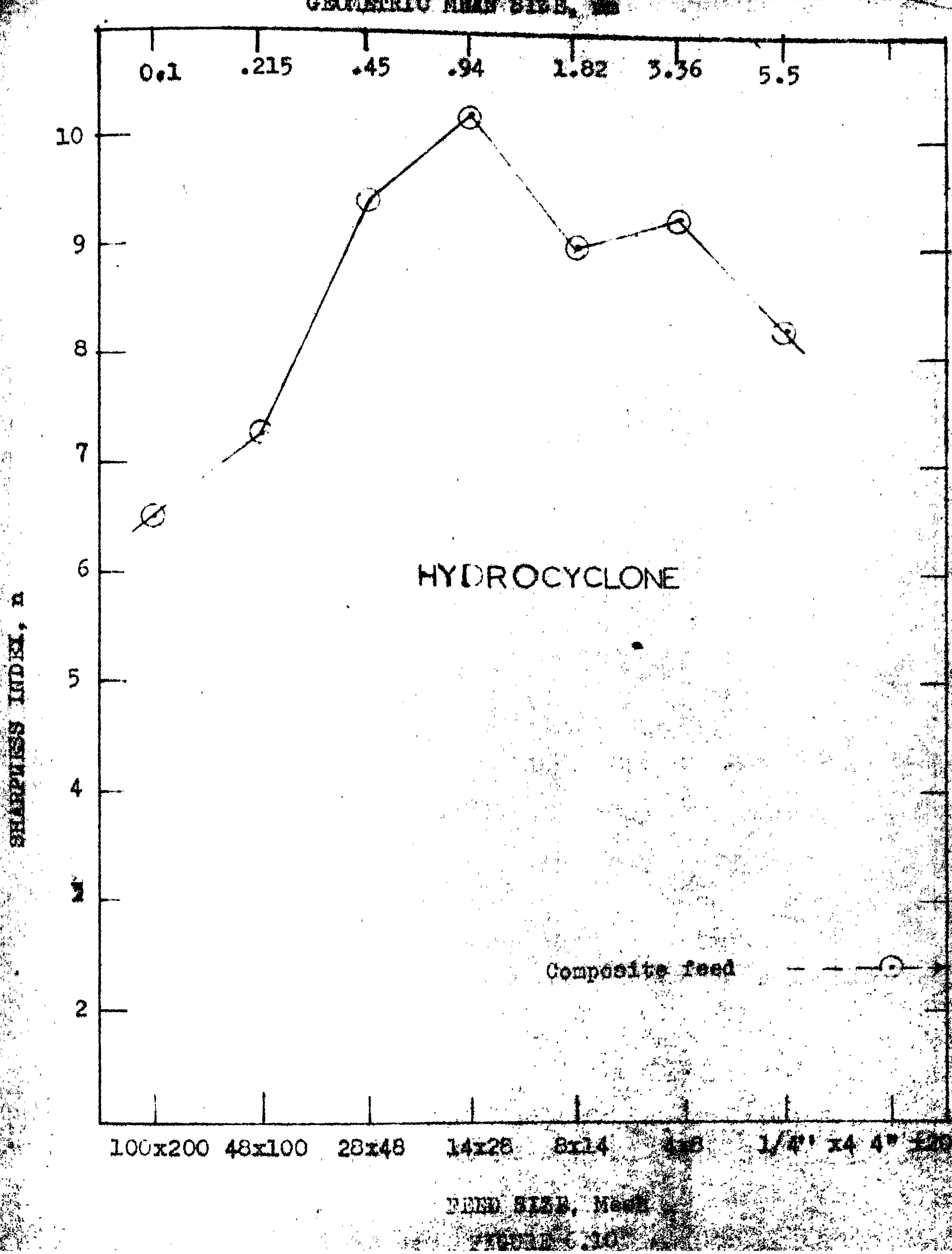

GEOMETRIC MEAN SIZE, mm
0.1
.215
.45
.94
1.82
3.36
5.5
SHARPNESS INDEX, n
10
9
8
7
6
5
4
3
2
HYDROCYCLONE
Composite feed
100x200
48x100
28x48
14x28
8x14
4x8
1/4" x4
4" x20
FEED SIZE, Mesh

FIGURE [illegible].10

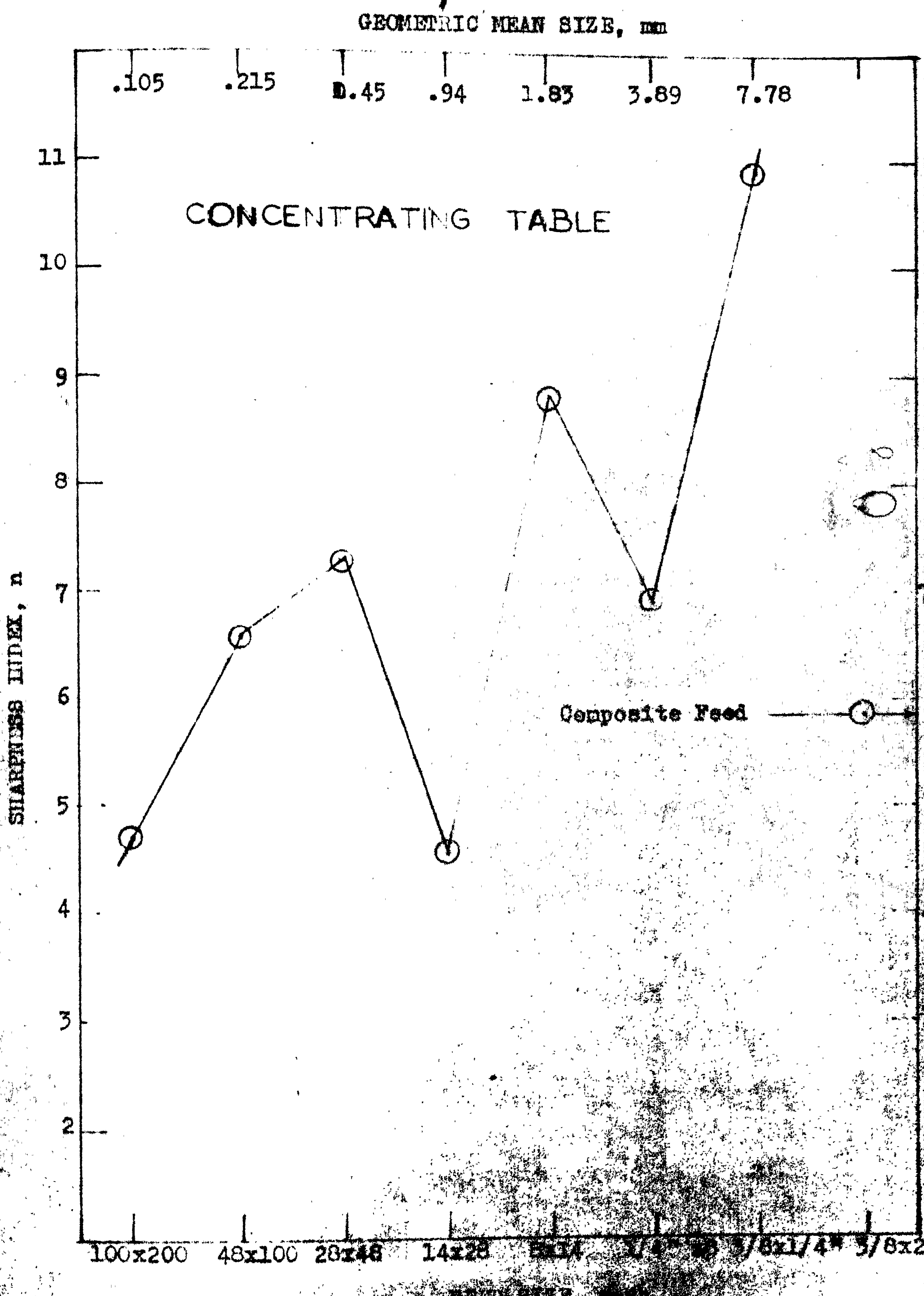
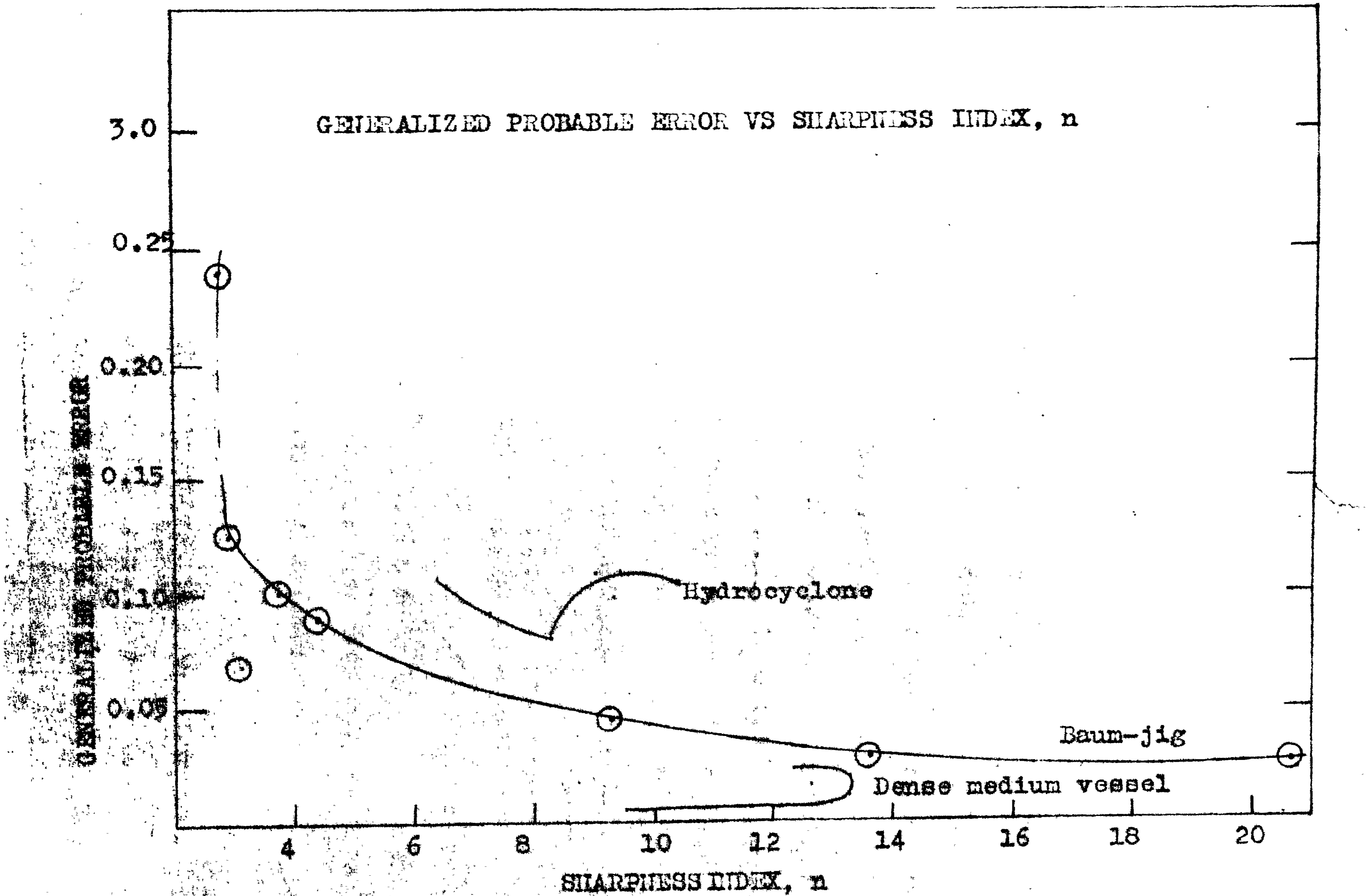

GEOMETRIC MEAN SIZE, mm
.105
.215
0.45
.94
1.83
3.89
7.78
11
10
9
8
7
6
5
4
3
2
CONCENTRATING TABLE
Composite Feed
SHARPNESS INDEX, n
100x200
48x100
28x48
14x28
3/8x1/4"
3/8x2

FIGURE 6.11


GENERALIZED PROBABLE ERROR VS SHARPNESS INDEX, n
GENERALIZED PROBABLE ERROR
3.0
0.25
0.20
0.15
0.10
0.05
Hydrocyclone
Baum-jig
Dense medium vessel
4
6
8
10
12
14
16
18
20
SHARPNESS INDEX, n

CHAPTER 7

ANALYSIS

7.1 Dense Medium Vessel

The sharpness index n varies from 12.3 to 13.3 for feeds in size ranges 1/2" x 1/4" to 4" x2" as given in Figure 6.6. Raw coal feed of sizes lower than 1/4" and greater than 4" have lower sharpness index. The composite feed of size 1/4" x 6" has a rather high sharpness index showing the ability of dense medium vessel to handle wide size range of raw coal feed. On the otherhand, the inability of dense medium vessels to handle fine size is evident from the droping nature of sharpness index curve below 1/2" size. These conclusions regarding dense medium vessels are in conformity with the literature on dense medium vessel performance [9a].

$f_u(\bar{d})$ - the maximum washed coal percentage that can be expected varies from 96.5 to 99.9 and $f_1(\bar{d})$ - the minimum coal in sink varies from 0.11×10^{-5} to 3.03 per cent. The value of $\bar{d}_1$, the limiting reduced density varies from 0.75 to 0.94 indicating that there is no difficulty in cleaning.

7.2 Baum Jig

In agreement with the literature [9b], the sharpness index increases with increase in feed size as given in Figure 6.7. It varies from 2.8 to 20.6 over a size range 14 x 28 mesh to 6" x3" and for a composite feed of size

$\bar{d}_1$ varied from 0.76 to 0.9, even difficult to wash coals can be handled by this process.

7.4 Hydrocyclone

The variation of sharpness index n with feed size is given in Figure 6.10. Here the sharpness index varies from 6.5 to 10.22, and is lower than for dense medium cyclone. The speciality of hydrocyclone in treating relatively fine 14 x 28 mesh size feed is evident by the parameter value of $n = 10.2$. Raw coal of feed size greater or less than this 28 mesh size has lower sharpness index. The composite feed of size 1/4" x 200 mesh has a very low value i.e. $n = 2.39$. Clearly hydrocyclone is found to be unsuitable for broad size range of raw coal feed.

Here $f_u(d)$ varied from 92.6 to 99.88 indicating somewhat less recovery than dense medium cyclone. The $f_1(\bar{d})$ varied from 0.005 to 10.9 indicating that under certain conditions more than 10 per cent of valuable coal may be lost to sink by this process. The variation of $\bar{d}_1$ from 0.11 to 0.5 show that hydrocyclones are not suitable for difficult to clean coals.

7.5 Concentrating Table

As shown in Figure 6.11, the sharpness index, n fluctuates sharply from 4.56 to 10.92. The underlying principles of concentrating table are somewhat different from the equipment considered earlier. It is not clear if our

model has a meaningful relevance to concentrating tables.

In any case, here $f_u(d)$ varies from 97.5 to 99.99 indicating higher recoveries of wasted coal, however, $f_1(\bar{d})$ values sometimes rise as high as 18.4, showing the unreliable nature of this process.

7.6 Generalized Probable Error:

The conventional probable error is given in Fig.2.2. In the case of generalized distribution curve as in Figure 2.4, we will have generalized probable error (GE_p) which can be simply written as

$$GE_p = 0.5\,[\bar{d}_{25} - \bar{d}_{75}]$$

where $\bar{d}_{25}$ is the reduced density at which 25 per cent of feed coal will report to clean coal product and $\bar{d}_{75}$ is the reduced density at which 75 per cent of feed coal will report to clean coal product.

Using the numerical values of the 4 parameters of our model, generalised probable error for all feed size fractions were calculated and tabulated in Appendix 4. A plot of the generalized probable error values vs sharpness index, n is given in figure 6.12.

CHAPTER 8

CONCLUSIONS

1. A mathematical model of the coal cleaning process has been derived which generates the separation (Tromp) curve. The 4-parameter model reproduces the distribution curves with a high degree of accuracy.

2. Rosenbrock hill climb method with uniform weights to the washed coal recovery $f(\bar{d})$ data, is suitable for estimating the 4 parameters in our model.

3. The model predicts self-similar distribution curves in $d/d_{0.5}$, in agreement with the plant data.

4. The model reproduces the distribution curves better than Gottfried's mathematical representation of distribution curves by the Weibull function.

5. The estimated values of the 4 parameters have been analyzed for all coal cleaning equipments. Higher the value of sharpness index, n, more efficient is the process. For better performance, n should be high, $f_u(\bar{d})$ should be maximum $f_l(\bar{d})$ should be minimum and $\bar{d}_l$ should be high.

6. Except for concentrating table valuable insight into the performance behaviour of all other float-sink equipment has been obtained, specially with respect to the important coal feed size variable.

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9(a) Palowitch E.R. and Deurbrouck,A.W.,Wet Concentration of Coarse Coal, Part 1- Dense Medium Separation, In: Ref.9 page 9-3.

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Coal Cleaning Devices, International Journal of Mineral Processing, 5 (1978) 1-20.

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APPENDIX 1

BOX-COMPLEX TECHNIQUE[14-15]

This method is based on the flexible geometric simplex method. Given 'N' independent variables method requires 'N+1' feasible starting cases (initial guesses). All the 'N+1' cases are randomly generated by varying the initial variable values between the upper and the lower bounds and are checked against the constraints. In this work for 4-parameter models, 5 starting cases randomly generated from the initial guess point.

Once the starting cases are provided, the search phase begins by evaluating the objective function, in our case at each of 5 points, and the worst case (for which the error is maximum) is rejected. The centroid of the remaining cases is found out and the rejected point is reflected on the opposite side of the centroid by a factor of GMR, reflection coefficient

$$X_{i,j(new)} = GMR\ (\bar{X}_{i,c} - \bar{X}_{i,j}\ (rejected)) + \bar{X}_{i,c}$$

where $X_{i,j}$ is the new point of the ith parameter

$$i = 1,2,\ldots,N$$

The value of GMR is arbitrarily chosen as 1.1. If the new case violates the constraints, it is modified by either GME, expansion coefficient or GMC, contraction coefficient, so that the new case is moved towards the centroid. This

procedure of moving towards the centroid is continued until the constraints are no more violated. After every iteration the objective function is checked for convergence until it is achieved. Figure 7 is the flow chart for the program used.

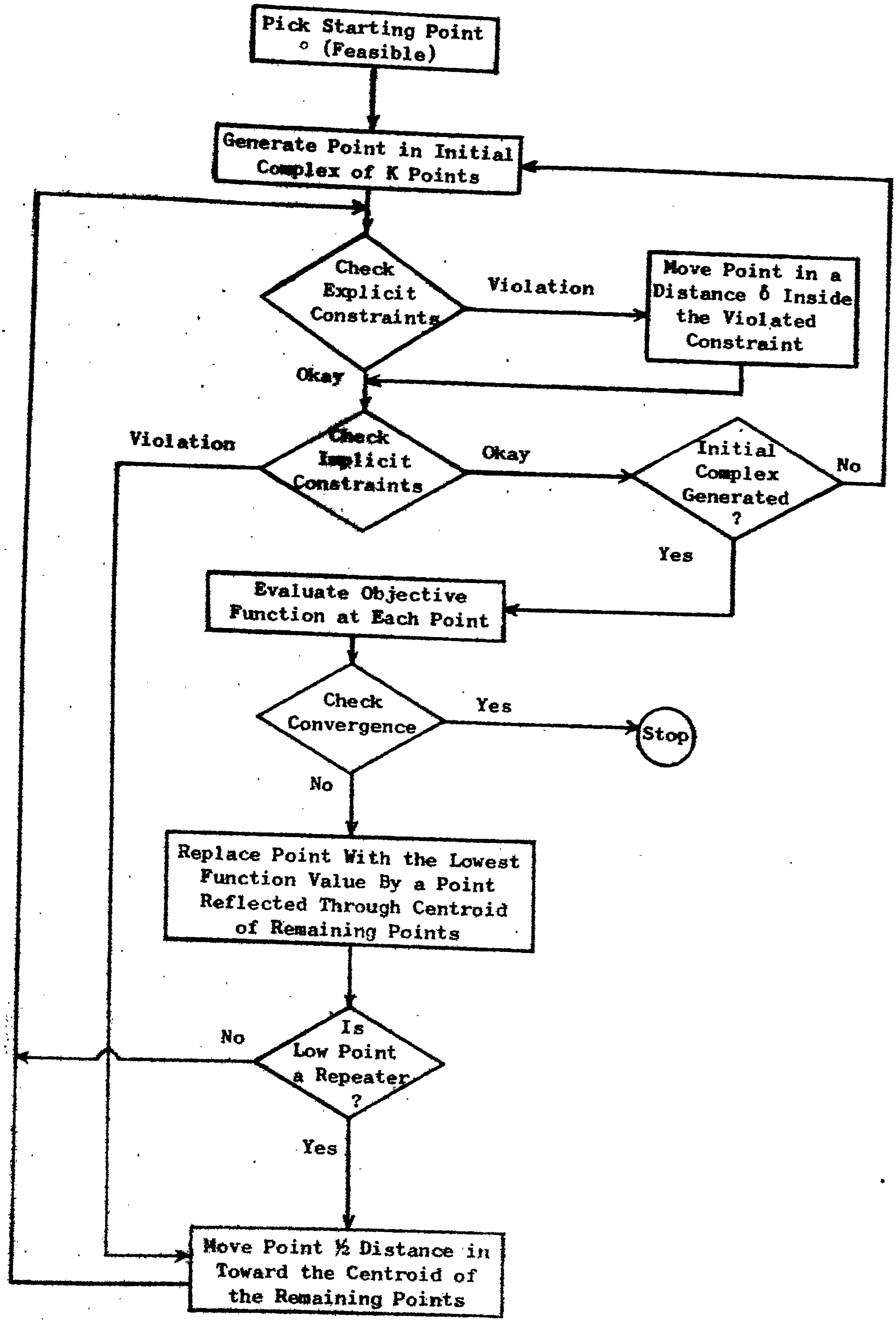
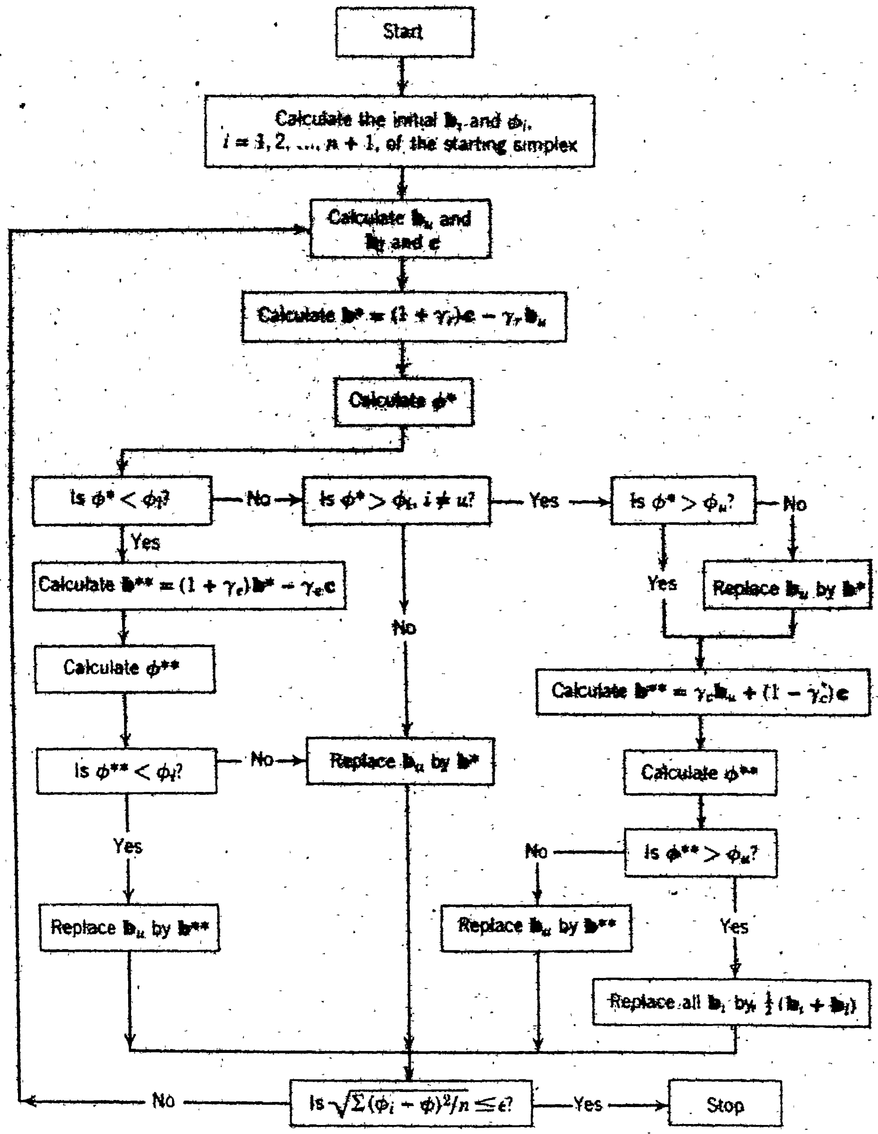


Figure 1. Box (COMPLEX ALGORITHM) Logic Diagram



7.2 Information flow chart for flexible simplex method.

```
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C     NP       : NUMBER OF PARAMETERS
C     NBR      : NUMBER OF PARAMETERS + 1
C     NPP1     : NUMBER OF READINGS.
C     D        : EXPT. VALUE OF INDEPENDENT VARIABLE(D).
C     FD       : EXPT. VALUE OF THE FUNCTION (F(D)).
C     B        : MATRIX CONTAINING PARAMETERS OF N+1 POINTS.
C     BB       : ARRAY CONTAINING PARAMETERS AT THE CURRENT POINT.
C     BU       : ARRAY CONTAINING PARAMETERS AT THE MAXIMUM PHI.
C     BL       : ARRAY CONTAINING PARAMETERS AT THE MINIMUM PHI.
C     BS       : ARRAY CONTAINING PARAMETERS OF THE REFLECTED POIN
C     BDS      : ARRAY CONTAINING PARAMETERS AFTER EXPANSION,CONTRA
C     C        : ARRAY CONTAINING PARAMETERS OF CENTROID.
C     PHI      : FUNCTION SUBPROGRAM TO CALCULATE ERROR FUNCTION.
C     PEY      : ARRAY CONTAINING ERROR(PHI) AT N+1 POINTS.
C     PHIS     : FUNCTION VALUE AT THE REFLECTED POINT.
C     PHI      : FUNCTION VALUE AT THE CENTROID.
C     PHIDS    : FUNCTION VALUE AT THE UPGRADED BU.
C     PHIMAX   : FUNCTION VALUE AT BU
C     PHIMIN   : FUNCTION VALUE AT BL.
C     GMR      : REFLECTION COEFFICIENT ~1.01
C     GME      : EXPANSION COEFFICIENT ~1.2
C     GMC      : CONTRACTION COEFFICIENT ~0.9
C     EPSILN   : EPSILON~0.1E-06
C     BB(1)    : DL
C     BB(2)    : M(SHARPNESS OF SEPARATION)
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      DIMENSION B(11,10),BB(10),BU(10),BL(10),BS(10),BDS(10),C(10
     1 ),D(  ),FD(  ),PEY(50),G(10),H(10),AZR(50,2,10),NP(10)
C     $$$$$$$$$$$$$$$       READ DATA          $$$$$$$$$$$$$$$$$$$
      READ(1,*)   ,NOEX
      DO   I=1,
      READ(1,*)FD(I),D(I)
C     $$$$$$$$$$$$$       READ COEFFICIENTS FOR BOX COMPLEX     $$$$$$
      READ(1,*)GMR,GME,GMC,EPSL
      READ(1,*)NP
C     $$$$$$$$$$$$$$$$       READ CONSTRAINTS        $$$$$$$$$$$
      DO   J=1,NP
      READ(1,*)G(J),H(J)
      DO 10 LTZZ=1,2
      LTZ=LTZZ
      NP=LTZ*2
      NPP1=NP+1
      NBR=1
      H(1)=G(1)
      READ(1,*)(B(1,J),J=1,NP)
      CALL GORTKP(G,H,NP,B,NPP1)
      DO 20 I=1,NPP1
      DO 11 K=1,NP
   11 BB(K)=B(I,K)
   20 PEY(I)=PHI(LTZ,BB,FD,D,NBR)
      CALL MINMAX(PEY,NPP1,PHIMAX,PHIMIN,JMIN,JMAX)
C     $$$$$$$$$$$$$$$$       CENTROID CALCULATION        $$$$$$$$$$
      DO 14 I=1,NP
      C(I)=0.
      DO 15 J=1,NPP1
      IF(J.EQ.JMAX)GOTO 15
      C(I)=C(I)+B(J,I)
   15 CONTINUE
      C(I)=C(I)/(NPP1-1)
   14 CONTINUE
      DO 12 J=1,NP
      BU(J)=B(JMAX,J)
   12 BL(J)=B(JMIN,J)
C     $$$$$$$$$$$$$$$$$$$       REFLECTED POINT      $$$$$$$$$$$$$$$$$
      DO 16 I=1,NP
```

```
16          BS(J)=(1.+GMR)*C(J)-GMR*BU(J)
            CALL CHKRFL(BS,NP,G,H,C)
            PHIS=PHI(LTZ,BS,FD,D,NBR)
            IF(PHIS.GE.PHIMIN)GOTO 201
C           $$$$$$$$$$$$$$$$$$$     EXPANSION          $$$$$$$$$$$$$$$$$$$
            DO 17 J=1,NP
17          BDS(J)=(1.+GME)*BS(J)-GME*C(J)
            CALL CHKRFL(BDS,NP,G,H,C)
            PHIDS=PHI(LTZ,BDS,FD,D,NBR)
            IF(PHIDS.GE.PHIMIN)GOTO 202
            DO 18 J=1,NP
18          BU(J)=BDS(J)
            GOTO 401
201         DO 19 J=1,NP
            IF(J.EQ.JMAX)GOTO 19
            IF(PHIS.GT.PFY(J))GOTO 19
            GOTO 202
19          CONTINUE
            GOTO 203
202         DO 21 J=1,NP
21          BU(J)=BS(J)
            GOTO 401
203         IF(PHIS.GT.PHIMAX)GOTO 204
            DO 22 J=1,NP
22          BU(J)=BS(J)
C           $$$$$$$$$$$$$$$$$$$     CONTRACTION        $$$$$$$$$$$$$$$$$$$
204         DO 23 J=1,NP
23          BDS(J)=GMC*BU(J)+(1.-GMC)*C(J)
            CALL CHKRFL(BDS,NP,G,H,C)
            PHIDS=PHI(LTZ,BDS,FD,D,NBR)
            IF(PHIDS.GT.PHIMAX)GOTO 205
            [illegible] J=1,[illegible]
[illegible] [illegible])=[illegible](J)
            [illegible] I=1,[illegible]
            [illegible] J=1,[illegible]
[illegible] [illegible](I,J)=0.5*(B(I,J)+BL(J))
[illegible] [illegible] J=1,NP
[illegible] [illegible],J)=BDS(J)
[illegible] [illegible] 31 J=1,NP
[illegible] [illegible],J)=BU(J)
[illegible] [illegible]=PHI(LTZ,C,FD,D,NBR)
            S[illegible]=0.
            [illegible] 27 J=1,NPP1
27          S[illegible]=S[illegible]+(PFY(J)-PHIC)**2/FLOAT(NPP1-1)
            S[illegible]=SQRT(SUM)
            IF(SU[illegible].LT.EPSLN) GOTO 206
            K[illegible]T=KOUNT+1
            IF(KOUNT.GT.LOOPX)GO TO 206
            GOTO 500
206         CALL PAPER(NP,C,H)
            TYPE 90,(J,B(JMIN,J),J=1,NP)
90          FORMAT(10X,'Y(',I1,')=',F15.8)
            DO 814 J=1,NP
814         C(J)=B(JMIN,J)
            TYPE 91,KOUNT
91          FORMAT(10X,'NO. OF STEPS=',I6)
            SRFA=0.
            F2R =0.
            DO 28 J=1,NBR
            GO TO (811,812),LIZ
811         FDC=FC2(C,NP,D(J))
            GOTO 816
812         FDC=FC4(C,NP,D(J))
816         AZB(J,1,1)=D(J)
            AZB(J,2,1)=FD(J)
            AZB(J,1,LIZ+1)=D(J)
            AZB(J,2,LIZ+1)=FDC
```

```
          ERRA=ERRA+(FDC-FD(J))**2
          ERRN=ERRN+((FDC-FD(J))/FD(J))**2
26        CONTINUE
          ERRA=ERRA/NBR
          ERRN=ERRN/NBR
          TYPE 93,ERRA,ERRN
93        FORMAT(1X,50(1H*),/,5X,'ERRA=',E15.8,5X,'ERRN=',E15.8,//)
3210      CONTINUE
          TYPE 257
257       FORMAT(//10X,'D',8X,'FD',8X,'FD2',8X,'FD4',/,2X,41(1H-))
          DO 259 JF=1,NBR
259       TYPE 258,AZB(JF,1,1),(AZB(JF,2,JG),JG=1,3)
258       FORMAT(6X,F7.4,4X,F6.2,5X,F6.2,5X,F6.2)
          NW(1)=NBR
          NW(2)=NBR
          STOP
          END
C         $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
          SUBROUTINE MINMAX(A,N,XMAX,XMIN,JMIN,JMAX)
          DIMENSION A(50)
          XMAX=A(1)
          XMIN=A(1)
          JMAX=1
          JMIN=1
          DO 10 I=2,N
          IF (XMAX.GE.A(I)) GOTO 20
          XMAX=A(I)
          JMAX=I
20        IF (XMIN.LE.A(I)) GOTO 10
          XMIN=A(I)
          JMIN=I
10        CONTINUE
          RETURN
          END
C         $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
          [illegible](LIO,BB,FD,D,NB)
          DIMENSION BB(10),FD(100),D(100)
          RAT=0.
          DO 10 I=1,NB
          W=(50.-ABS(FD(I)-50.))/50.
          W=1.
          GOTO (951,952),LIO
951       FDC=100./(1.+((D(I)-BB(1))/(1.-BB(1)))**BB(2))
          GOTO 11
952       FDC=BB(4)+(BB(3)-BB(4))/(1.+((BB(3)-50.)/(50.-BB(4)))*(((D(I)
         1 -BB(1))/(1.-BB(1)))**BB(2)))
11        RAT=RAT+W*(FDC-FD(I))**2
          RAT=RAT/NB
          RETURN
          END
C         $$$$$$$$$$        GENERATION OF K-1 POINTS USING RANDOM NOS. $
          SUBROUTINE GNKTKP(G,H,NP,B,NPP1)
          DIMENSION G(10),H(10),B(11,10)
          DO 10 I=2,NPP1
          DO 10 J=1,NP
          DO 15 JJ=1,7
15        X=RAN(X)
10        B(I,J)=G(J)+X*(H(J)-G(J))
          RETURN
          END
C         $$$$$$$$        CHECKS WHETHER THE REFLECTED POINT XR VIOLATE
C                               S THE CONSTRAINTS OR NOT      $$$$$$$$$$$$

          SUBROUTINE CHKREF(XR,NP,G,H,C)
          DIMENSION XR(10),G(10),H(10),C(10)
          DO 10 I=1,NP
          IF(XR(I).LT.H(I).AND.XR(I).GT.G(I))GOTO10
          GOTO 11
10        CONTINUE
          GOTO 80
```

```
11          DO 12 J=1,NP
12          YR(J)=(XR(J)+C(J))*0.5
            GOTO 14
9           RETURN
            END
            SUBROUTINE MPLOT(A,NASA2,NASA,NLY,NLXX)
C           ************************************************
C
C                   A      THIS A 3 DIMENSIONAL MATRIX WHOSE ELEMENTS
C                               ARE AS FOLLOWS:
C           A(I,1,K)=X-CO ORDINATE
C                               A(I,2,K)=Y-CO ORDINATE
C                               I=1,50(NO. OF POINTS)
C                               K=INTEGER WHOSE VALUE DETERMINED
C             BY THE POINT.(TO WHICH GRAPH IT BELONGS
C                                                                          TO)
C                   NASA2:     NUMBER OF GRAPHS TO BE PLOTTED
C                   NASA :     AN ARRAY CONTAINING NUMBER OF POINTS
C                              IN EACH GRAPH.
C                   NLY  :     NUMBER OF LINES IN Y-AXIS
C                   NLX  :     NUMBER OF LINES IN X-AXIS.
C
C           ************************************************
            DIMENSION X(1000),Y(1000),SX(11),SY(11),ILC(120)
            DIMENSION A(50,2,10),NASA(10),ISYMB(10),IMY(1000)
            DIMENSION ILB(120),ILL(120),YXX(1000),ILD(200)
            DATA IM,IC,IB,IX,IZ/'-','!',' ','*',1H!/
            DATA ISYMB(1),ISYMB(2),ISYMB(3)/'1','2','3'/
            DATA ISYMB(4),ISYMB(5),ISYMB(6),ISYMB(7)/'4','5','6','7'/
            DATA ISYMB(8),ISYMB(9),ISYMB(10)/'8','9','0'/
            NLX=NLXX
            NYCN=7.5
75          FORMAT(1H1)
            N=0
            DO 311 K=1,NASA2
            DO 310 J=1,NASA(K)
            N=N+1
            Y(N)=A(J,2,K)
            X(N)=A(J,1,K)
310         IMY(N)=K
311         CONTINUE
            DO 1 I=1,N
            II=I
            DO 2 J=II,N
            IF(Y(I).GT.Y(J))GOTO3
            GOTO 2
3           TEMP=Y(I)
            Y(I)=Y(J)
            Y(J)=TEMP
            TEMP=X(I)
            X(I)=X(J)
            X(J)=TEMP
            ITEMP=IMY(I)
            IMY(I)=IMY(J)
            IMY(J)=ITEMP
2           CONTINUE
1           CONTINUE
            XMIN=X(1)
            XMAX=X(1)
            KCHECK=0
            DELY=(Y(1)-Y(N))/(NLY-1)
            DO 4 I=1,N
            IF(X(I).GT.XMIN)GOTO6
            XMIN=X(I)
6           IF(X(I).LT.XMAX)GOTO4
```

```
93        FORMAT(35X,'DELY=',E15.8,10X,'DELX=',E15.8/)
          JX=NLX/10+1
          JY=NLY/5+1
          DO 5 I=1,JX
5         SX(I)=XMIN+DELX*10.*(I-1)
          DO 8 I=1,JY
8         SY(I)=Y(1)-DELY*5.*(I-1)
          DO 60 I=1,NLX
          ILD(I)=ID
          ILB(I)=IB
60        ILC(I)=IB
          DO 7 I=2,NLX
          M=I-(I/10)*10
          IF(M.NE.1)GOTO 9
          ILL(I)=IP

          GOTO7
9         ILL(I)=ID
7         CONTINUE
          I[illegible]C(1)=IP
          I[illegible]C([illegible]LY)=IP
          I[illegible](1)=IP
          I[illegible]([illegible]Y)=IP
          I[illegible](1)=IP
          I[illegible](1)=IP
          I[illegible]([illegible]Y)=IP
          [illegible]2,(SX(I),I=1,JX)
          [illegible]1,(I[illegible]L(I)),I=1,NLX)
[illegible]   [illegible](15X,11(F9.3,1X))
[illegible]   [illegible](2[illegible]X,102A1)
[illegible]   [illegible](' [illegible]8450')
          I[illegible]([illegible]*X[illegible]AX.GE.0)GOTO16
          [illegible]Z=-([illegible]/DELX
          [illegible]=L
1[illegible]  [illegible]1=[illegible]
          Y([illegible]+1)=Y([illegible])-DELY
          [illegible]I=1,[illegible]
          [illegible]=([illegible](I)-XMIN)/DELX+1
          [illegible]Y=(Y(1)-Y(I))/DELY+0.5
          [illegible]Y1=(Y(1)-Y(I+1))/DELY+0.5
          [illegible]=[illegible]Y1-IY
          I[illegible]([illegible]K-1)61,62,63
61        ILC([illegible]X)=ISY[illegible]B(IMY(I))
          GOTO16
63        [illegible]=[illegible]K-1
          I[illegible]C([illegible]X)=ISY[illegible]B(IMY(I))
          [illegible]P1=[illegible]Y-([illegible]Y/5)*5
          [illegible]P2=[illegible]Y/5+1
          I[illegible](KCHECK.NE.1)GOTO300
          I[illegible]C([illegible]ZZ)=IZ
3[illegible]      IF([illegible]P1.[illegible]E.0)GOTO303
          IF([illegible]Y.EQ.NLY .OR.[illegible]Y.EQ.0)GOTO302
          DO 301 JBT=1,NLX
          IF(ILC(JBT).NE.IB)GOTO301
          I[illegible]([illegible]T)=IP
301       CONTINUE
302       TYPE 96,SY(NP2),(ILC(KX),KX=1,NLX)
          GOTO304
303       TYPE 97,(ILC(KL),KL=1,NLX)
C         ****************************************************
304       DO 65 KP=1,[illegible]K
          [illegible]P1=([illegible]Y+KP )/5.+1
          [illegible]P=[illegible]Y+KP -([illegible]P1-1)*5
          I[illegible]([illegible]P.NE.[illegible])GOTO71
          I[illegible](KCHECK.NE.1)GOTO307
          IL[illegible](NZZ)=IZ
```

```
      CONTINUE
 97   FORMAT(20X,102A1)
      GOTO195
C     ***********************************
 52   CONTINUE
      ILC(NX)=ISYMB(IMY(I))
      NP1=NY-(NY/5)*5
      NP2=NY/5+1
      IF(KCHECK.NE.1)GOTO78
      ILC(NZZ)=IZ
 78   IF(NP1.NE.0)GOTO73
      IF(NY.EQ.NLY-1 .OR.NY.EQ.0)GOTO83
      DO 74 JBT=1,NLX
      IF(ILC(JBT).NE.IB)GOTO74
      ILC(JBT)=ID
 74   CONTINUE
 83   TYPE 96,SY(NP2),(ILC(KX),KX=1,NLX)
      GOTO195
 73   TYPE 97,(ILC(KL),KL=1,NLX)
 96   FORMAT(5X,F15.8,102A1)
C     ********************************
[illegible]
      [illegible]=1,[illegible]
      [illegible]=1
      [illegible])=I[illegible]
      [illegible])=I[illegible]
[illegible]
      [illegible],([illegible],I=1,[illegible])
      [illegible],([illegible]),I=1,[illegible])
      [illegible]=1,[illegible]
 7[illegible]  X([illegible])=[illegible](ITK)
[illegible]
[illegible]
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      [illegible]2(X,N,DXP)
      [illegible] X([illegible])
      [illegible]=1[illegible]./(1.+((DXP-X(1))/(1.-X(1)))**X(2))
[illegible]
[illegible]
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      [illegible]4(X,N,DXP)
      [illegible] X(4)
      [illegible]=X(4)+(X(3)-X(4))/(1.+((X(3)-50.)/(50.-X(4)))*(((DXP-X(
     1)/(1.-X(1))))**X(2)))
[illegible]
[illegible]
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      [illegible] D[illegible] (P,G,H)
      [illegible] G(10),H(10)
      [illegible]
      TYPE 253,P
 253  [illegible](10X,45(1H$),/,20X,I1,'PARAMETER MODEL',/10X,45(1H$)
      TYPE 254
 254  [illegible](23X,'CONSTRAINTS:',/,23X,12(1H-)/)
      [illegible] 10 J=1,P
      ZG=G(J)
      ZA=H(J)
 10   TYPE 255,ZG,J,ZA
 255  [illegible](20X,F6.2,'< X(',I1,') <',F6.2)
      [illegible]
      [illegible]
```

APPENDIX 2

ROSENBROCK HILL CLIMB METHOD [16-17]

Unlike the Box-Complex technique this method requires only one feasible starting case (Initial guesses). In this method the initial guesses is chosen and the appropriate constraints are posed on all the variables. Once the feasible starting case is given, initial step sizes, $E_i (i=1,2,...,N)$ are picked up and the objective function is evaluated. Then the first variable $\bar{\bar{X}}_1$ is stepped a distance 'E_1,' parallel to the axis and the objective function is evaluated. The current best objective function is defined by a value of F_0 where the constraints are satisfied and 'H' the current best objective function value for a point where the constraints are satisfied in addition to the boundary zones are not violated. 'H' and 'F_0' are initially set equal to the objective function value at the starting point. If in the current point objective function evaluation, H is worse than F_0, or if the constraints are violated, then the trial is a failure and the value of 'E_1', is decreased by a factor of 0.5 and the direction of movement is reversed. Also if there is function improvement then E_1 is increased by three times its original value and the search is carried out in the next direction.

On the otherhand, if the current point lies within a boundary zone, the objective function is modified as follows:

$$F_{(new)} = F_{(old)} - (F_{old} - H)\ (3\ P_W - 4P_W^{\ 2} + 2\ P_W^2)$$

where $$P_W = \frac{G_K + (H_K - G_K)\ 10^{-4} - X_K}{(H_K - G_K)\ 10^{-4}}$$

For lower zone, where G_K is lower constraint and H_K is upper constraint.

and $$P_W = \frac{X_K - (H_K - (H_K - G_K)\ 10^{-4})}{(H_K - G_K)\ 10^{-4}}$$

For upper zone,

At the inner edge of the boundary zone, $P_W=0$, that is the function is unaltered ($F_{(new)} = F_{(old)}$). Then the function value is replaced by the best current function value in the feasible region and not in the boundary zone. For the function which improves as the constraints is approached, the modified function has an optimum in the boundary zone.

After search in all the coordinate directions, it is checked for convergence. If convergence is not achieved then it is checked for one success and one failure in each direction and if it is not so then the whole procedure is repeated. On the other hand, if one success and one failure is achieved in each direction, then the axes are rotated according to the equation,

$$M_{i,j}(K+1) = \frac{D_{i,j}\ (K)}{\left[\sum_{l=1}^{N} (D_{l,j}^{(K)}\ )^2\right]^{1/2}}$$

where $D_{i,1}^{(K)} = A_{i,1}^{(K)}$

$$D_{i,j}^{(K)} = A_{i,j}^{(K)} - \sum_{l=1}^{j-1} \left[\sum_{n=1}^{i} M_{n,l}^{(K+1)} \right]$$

$$A_{i,j}^{(K)} = \sum_{l=j}^{N} d_i^{(K)} \quad M_{i,l}^{(K)} \quad , \; j = 2,3,\ldots,N$$

Also i = variable index ($i = 1,2,\ldots.,N$)

j = direction index ($j = 1,2,\ldots,N$)

K = stage index

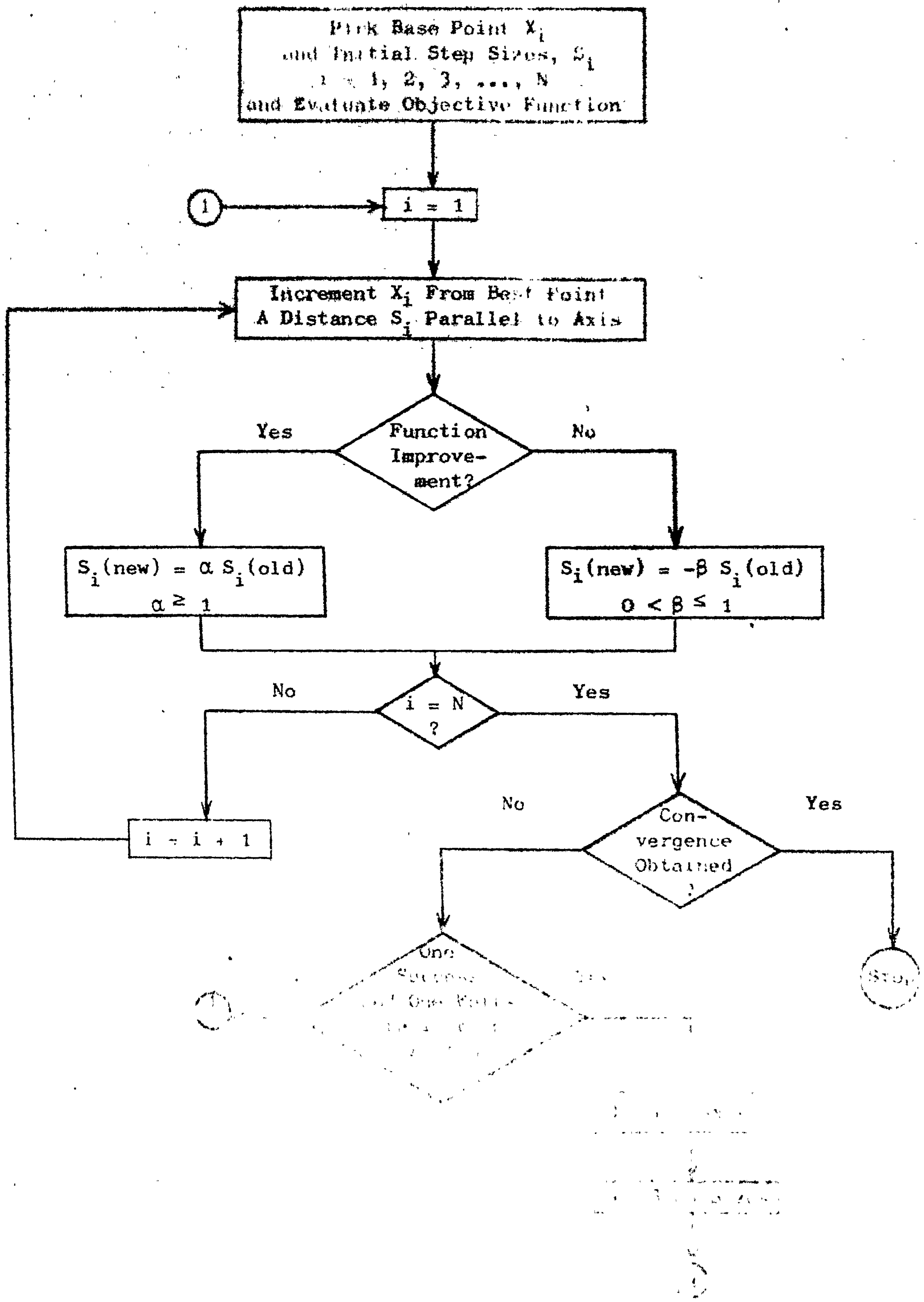
d_i = sum of distance moved in the 'i' direction since last rotation of axes.

$M_{i,j}$ = direction vector component normalized.

After rotating the axes the search is made in each of the x direction using the new coordinate axis.

$$X_{i(new)}^{(K)} = X_{i(old)}^{(K)} + E_j^{(K)} \quad M_{i,j}^{(K)}$$

The above procedure is repeated until convergence is achieved. Figure 8 is the flow chart for the program used.


Pick Base Point X_i
and Initial Step Sizes, S_i
$i = 1, 2, 3, \ldots, N$
and Evaluate Objective Function
1
$i = 1$
Increment X_i From Best Point
A Distance S_i Parallel to Axis
Yes
Function Improvement?
No
$S_i(\text{new}) = \alpha\, S_i(\text{old})$
$\alpha \geq 1$
$S_i(\text{new}) = -\beta\, S_i(\text{old})$
$0 < \beta \leq 1$
No
$i = N$?
Yes
$i = i + 1$
No
Con- vergence Obtained ?
Yes
Stop

```
C     ########  HILL 2&4 PARAMETER MODEL    #########
C     MATH LINE PROGRAM FOR ROSENBROCK HILL CLIMB
C
C     M          :PROBLEM CONTROLLER +1 FOR MAXIMIZATION,
C                      -1 FOR MINIMIZATION.
C     P          : NUMBER OF VARIABLES
C     L          : NUMBER OF CONSTRAINTS
C     K          : POINT INDEX
C     E          : VECTOR OF INITIAL STEP SIZES.
C     X          : VECTOR OF INITIAL GUESSES FOR INDEPENDENT VARIABLES.
C     N          :DIMENSION LIMIT IN SUBROUTINES.
C     V          : DIRECTION VECTOR
C     PR         : PRINTING CONTROLLER (NO. OF STAGES BETWEEN OUTPUTS.
C     ND         : STORAGE CONTROLLER
C     DA         : GENERAL STORAGE VECTOR.
C     DX         : INDEPENDENT VARIABLES OF THE DATA.
C     FD         : DEPENDENT VARIABLES OF THE DATA.
C     LIZ        : CONTROLLER ON THE MODEL 2 OR 4 PARAMETERS.
C     NDATA      : NUMBER OF DATA POINTS TO BE READ INTO DA.
C     NPAR       : DIMENSION LIMIT IN SUBROUTINES.
C     DELY       : ERROR IN OBJECTIVE FUNCTION.
C     NSTEP      : CONTROLLER ON STEP SIZES FOR EACH ROTATION
C                  =0 FOR ORIGINAL STEP SIZES,1 FOR STEP SIZES
C                  =FROM PREVIOUS ROTATION.
C     LOOPY      : MAXIMUM NO. OF STAGES TO BE CALCULATED.
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      [illegible] X(10),E(10),V(10,10),SA(10),D(10),G(10),H(10),AL(10),
     [illegible],[illegible](10,10),[illegible](10,10),DX(10),DA(20),VV(10,10),EINT(10),
     [illegible],[illegible],[illegible](50),AZB(50,2,10),[illegible](10),YV(10),EV(10)
      [illegible]
      [illegible] P,C,M,PR
      [illegible]
      [illegible] 12
    1 [illegible](3X,30HROSENBROCK HILLCLIMB PROCEDURE)
      READ(1,*) NDATA
      DO 201 JPX=1,NDATA
  201 READ(1,*) FD(JPX),DX(JPX)
      DO 3011 LIZ=1,2
      READ(1,*)M,P,L,LOOPY,PR,ND,NDATA,NSTEP
      READ(1,*)(X(K),K=1,P)
      READ(1,*)(E(J),J=1,P)
      CALL PAPER(P)
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      IF(ND-1)30,20,30
   20 READ(1,*)(DA(KA),KA=1,NDATA)
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
   30 [illegible]=PR-1
      [illegible]=[illegible]
      [illegible]=[illegible]
      [illegible]=[illegible]
      [illegible]=0
      [illegible]=[illegible]
      [illegible]=1.E-1[illegible]
      [illegible]=[illegible]
      [illegible]= DATA
      [illegible]=[illegible]
      DO 10 K=1,L
      KDF=K
   10 AL(K)=(CH(Y,DX,N,NPAR,KDF)-CG(Y,DX,N,NPAR,KDF))*0.0001
      DO 60 I=1,P
      DO 60 J=1,P
      V(I,J)=0.0
      IF(I-J)60,61,60
   61 V(I,J)=1.0
   60 CONTINUE
      DO [illegible] KK=1,P
      EINT(KK)=E(KK)
   [illegible] CONTINUE
  100 DO 70 I=1,P
```

```
      IF(ASIER.EQ.0)E(J)=EINT(J)
      SA(J)=2.0
7     D(J)=0.0
      FBEST=F1
      I=1
      IF(INIT.EQ.0)GO TO 120
      DO 110 K=1,P
110   X(K)=X(K)+E(I)*V(I,K)
      DO 50 K=1,I
5     H(K)=F0
120   GOTO (920,921),LTZ
920   F1=FM2(X,DA,N,NPAR,FD,DX)
      GOTO 922
921   F1=F(X,DA,N,NPAR,FD,DX)
922   F1=M*F1
      IF(ISW.EQ.0)F0=F1
      ISW=1
      IF(ABS(FBEST-F1)-DELY)122,122,125
122   TERM=1.0
      GOTO450
125   CONTINUE
      J=1
130   XC=CX(X,DX,[illegible],NPAR,J)
      LC=CG(X,DX,[illegible]PAR,J)
      UC=CH(X,DX,[illegible]PAR,J)
      IF([illegible]) [illegible] 420
      [illegible]
      [illegible] 420
      [illegible](J)) GOTO 140
      [illegible](J)) GOTO 140
      [illegible]
      [illegible]
      [illegible]
      [illegible](J)
      [illegible].LC.[illegible].UC.LE.XC) GOTO 150
      [illegible].XC.[illegible].XC.[illegible]C+B[illegible]) GOTO 160
      [illegible].XC.AND.XC.LT.UC) GOTO 170
      [illegible](J)=1.0
      [illegible] 21
      [illegible](J)=[illegible]
      [illegible] 1[illegible]
      [illegible]=([illegible]+B[illegible]-XC)/BW
      [illegible] 1[illegible]
1[illegible]  [illegible]=(XC-UC+B[illegible])/B[illegible]
1[illegible]  P[illegible](J)=1.0-3.0*P[illegible]+4.0*PW*PW-2.0+PW*PW*PW
1[illegible]  F[illegible]=[illegible](J)+(F1-H(J))*PH(J)
2[illegible]  C[illegible]TINUE
      IF([illegible].I) GOTO 220
      J=J+1
      GOTO 130
2[illegible]  I[illegible]=1
      IF(F1.[illegible]) GOTO 420
      [illegible](I)=[illegible](I)+E(I)
      [illegible](I)=3.[illegible]*E(I)
      [illegible]=1
      IF(SA(I).GE.1.5) SA(I)=1.0
2[illegible]  DO 2[illegible] JJ=1,P
      IF(S[illegible](JJ).GE.0.5) GOTO 440
2[illegible]  C[illegible]TINUE
C[illegible]$$$$$$[illegible]$        AXES ROTATION    $$$$$$$$$$$$$$$$$$$$
      DO 25[illegible] R=1,P
      DO 25[illegible] C=1,P
250   VV(C,R)=0.0
      DO 25[illegible] R=1,P
      KR=R
      DO 25[illegible] C=1,P
      DO 255 K=KR,P
255   VV(R,C)=D(K)*V(K,C)+VV(R,C)
25[illegible] B(R,C)=VV(R,C)
```

```
      BMAG=0.0
      DO 280 C=1,P
      BMAG=BMAG+B(1,C)*B(1,C)
280   CONTINUE
      BMAG=SQRT(BMAG)
      PX(1)=BMAG
      DO 310 C=1,P
310   V(1,C)=B(1,C)/BMAG
      DO 340 R=2,P
      IR=R-1
      DO 390 C=1,P
      SUMVM=0.0
      DO 320 KK=1,IR
      SUMAV=0.0
      DO 330 KJ=1,P
330   SUMAV=SUMAV+VV(R,KJ)*V(KK,KJ)
320   SUMVM=SUMAV*V(KK,C)+SUMVM
390   B(R,C)=VV(R,C)-SUMVM
      DO 340 R=2,P
      BBMAG=0.0
      DO 350 K=1,P
350   BBMAG=BBMAG+B(R,K)*B(R,K)
      BBMAG=SQRT(BBMAG)
      DO 340 C=1,P
340   V(R,C)=B(R,C)/BBMAG
      LOOP=LOOP+1
      LAP=LAP+1
      [illegible] 450
      [illegible] 450
      [illegible] IX=1,P
      [illegible] X(I)*[illegible](I,IX)
      [illegible]
      [illegible] SA(I)=0.0
      [illegible]
      [illegible]
      [illegible] 81
      I=I+1
      [illegible]
      [illegible] GOTO 898
      [illegible]
3     [illegible](/,15X,8HSTAGE,8X,9H[illegible],12X,8HPROGRESS,9X,
     1[illegible] PROGRESS)
      [illegible] LOOP,[illegible],BMAG,BBMAG
4     [illegible](/,I5,3F20.8)
      [illegible]
1[illegible] [illegible](15X,33H[illegible]UMBER OF FUNCTION EVALUATIONS = ,I8)
      [illegible]
5     [illegible](/,15X,25HVALUES OF X AT THIS STAGE)
C     $$$$$$ [illegible] CURRENT VALUES OF X $$$$$$$
      [illegible],(J[illegible],X(J[illegible]),J[illegible]=1,P)
6     [illegible](18X,2HX(,I2,4H ) =,F14.6)
      [illegible]
      [illegible]
      [illegible] 470
      [illegible] 480
      [illegible] 480
      [illegible] 1000
47    [illegible] 7
7     [illegible](/,2X,'THE STARTING POINT MUST NOT VIOLATE THE
     1[illegible] S[illegible]TS,IT APPEARS TO HAVE DONE SO.')
4[illegible] [illegible]
      [illegible]
      [illegible]
      TYPE 999
9[illegible] FORMAT(16X,63(1H-))
      TYPE 2002
2002  FORMAT(25X,' OX(I)' ,6X,' F[illegible](1)' ,8X,' [illegible]' ,9X, ' DIFF' )
      TYPE 999
      DO 2001 I=1,NPAR
```

```
      GOTO (961,962),LIZ
961   FCAL=FC2(X,N,DX(I))
      GOTO 963
962   FCAL=FC4(X,N,DX(I))
963   AZB(I,1,1)=DX(I)
      AZB(I,2,1)=FD(I)
      AZB(I,1,2)=DX(I)
      AZB(I,2,2)=FCAL
      ERRA=ERRA+(FCAL-FD(I))**2
      ERRN=ERRN+((FCAL-FD(I))/FD(I))**2
      ZDC=FCAL-FD(I)
2001  TYPE91,DX(I),FD(I),FCAL,ZDC
91    FORMAT(23X,4(F8.4,5X))
      NN(1)=NPAR
      NN(2)=NPAR
      ERRA=ERRA/FLOAT(NPAR)
      ERRN=ERRN/FLOAT(NPAR)
      TYPE 999
      TYPE 3001,ERRA,ERRN
3001  FORMAT(16X,'ABS.ERROR IS= ',F15.8,5X,'NOR.ERROR IS =',F15.8)
      TYPE 999
3210  CONTINUE
286   CONTINUE
      CALL [illegible](AZB,2,NN,21,51)
      STOP
      [illegible]
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      [illegible] FC( , , ,NPAR,FD,DX)
      [illegible] X( ), [illegible],FD(NPAR),DX(NPAR)
      [illegible]
      [illegible]
      [illegible] I=1,[illegible]
      [illegible]=( .-[illegible](FD(I)-50.))/50.
      [illegible]
      [illegible]=X( )+(X(3)-X(4))/(1.+((X(3)-50.)/(50.-X(4)))*(((DX(I)-X(1))
     1/( .-X(1)))**X(2)))
1     [illegible]*(FOC-FD(I))**2
      [illegible]
      [illegible]=[illegible]+1
      [illegible]
      [illegible]
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      FUNCTION CX(Y,DA,N,NPAR,K)
      DIMENSION Y( ),DA(NPAR)
      CX=X( )
      RETURN
      END
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      FUNCTION CG(Y,DA,N,NPAR,K)
      DIMENSION Y(N),DA(NPAR)
      GOTO (1,2,4,5),K
1     C[illegible]=
      [illegible]
2     C[illegible]=
      [illegible]
4     C[illegible]=[illegible].
      [illegible]
5     C[illegible]=
      [illegible]
      [illegible]
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      FUNCTION CH(Y,DA,N,NPAR,K)
      DIMENSION Y(N),DA(NPAR)
      GOTO (1,2,4,5),K
1     C[illegible]=[illegible].[illegible]
      [illegible] 3
2     C[illegible]=[illegible].
      [illegible]
4     C[illegible]=[illegible].
```

```
      GOTO 3
5     CN=50.
3     RETURN
      END
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      FUNCTION FM2(X,DA,N,NPAR,FD,DX)
      DIMENSION X(N),DA(NPAR),FD(NPAR),DX(NPAR)
      COMMON KOUNT
      FM2=0.0
      DO 1009 I=1,NPAR
      WW=(50.-ABS(FD(I)-50.))/50.
      WW=1.
      FDC=100./(1.+((DX(I)-X(1))/(1.-X(1)))**X(2))
      FM2=FM2+WW*(FD(I)-FDC)**2
1009  CONTINUE
      FM2=FM2/NPAR
      KOUNT=KOUNT+1
      RETURN
      END
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      FUNCTION FC2(X,N,DXP)
      DIMENSION X(N)
      FC2=100./(1.+((DXP-X(1))/(1.-X(1)))**X(2))
      RETURN
      END
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      [illegible]
      [illegible] X( )
      [illegible])+(X(2)-[illegible]))/(1.+((X(3)-50.)/(50.-X(4)))*(((DXP-X(1))
     */(1.-X(1)))**X(2)))
      [illegible]
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      [illegible] (P)
      [illegible]
      [illegible] 253,P
253   [illegible](23X,45(1H*),/,33X,I1,'PARAMETER MODEL',/23X,45(1H*)//)
      [illegible] 254
254   FORMAT(36X,'CONSTRAINTS:',/,23X,12(1H-)/)
      [illegible] I=1,N
      [illegible]
      ZG=[illegible](X,DX,[illegible],NPAR,IYP)
      ZA=[illegible](X,DX,[illegible],NPAR,IYP)
      [illegible] TYPE 255,ZG,I,ZA
255   FORMAT (20X,F6.2,'<X(',I1,')<',F6.2)
      [illegible]
      [illegible]
C     $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
      SUBROUTINE IPLOT(A,NASA2,NASA,NLY,NLXX)
C     ************************************************************
C
C            A      THIS A 3 DIMENSIONAL MATRIX WHOSE ELEMENTS
C                        ARE AS FOLLOWS:
C     A(I,1,K)=X-CO ORDINATE
C                             A(I,2,K)=Y-CO ORDINATE
C                     -       I=1,50(NO. OF POINTS)
C                             K=INTEGER WHOSE VALUE DETERMINED
C        BY THE PRINT.(TO WHICH GRAPH IT BELONGS
C                                                                   TO)
C
C            NASA2:      NUMBER OF GRAPHS TO BE PLOTTED
C            NASA :      AN ARRAY CONTAINING NUMBER OF POINTS
C                        IN EACH GRAPH.
C            NLY  :      NUMBER OF LINES IN Y-AXIS
C            NLXX :      NUMBER OF LINES IN X-AXIS.
C
C     ************************************************************
      DIMENSION X(1000),Y(1000),SX(11),SY(11),ILC(120)
      DIMENSION A(50,2,10),NASA(10),ISYMB(10),IMY(1000)
      DIMENSION ILB(120),ILL(120),XXX(1000),ILD(200)
```

```
      DATA ID,IP,IB,IX,IZ/'-','!',' ','*',1H!/
      DATA ISYMB(1),ISYMB(2),ISYMB(3)/'1','2','3'/
      DATA ISYMB(4),ISYMB(5),ISYMB(6),ISYMB(7)/'4','5','6','7'/
      DATA ISYMB(8),ISYMB(9),ISYMB(10)/'8','9','0'/
      NLX=NLXX
      TYPE705
705   FORMAT(1H1)
      N=0
      DO311K=1,NASA2
      DO310J=1,NASA(K)
      N=N+1
      Y(N)=A(J,2,K)
      X(N)=A(J,1,K)
310   IMY(N)=K
311   CONTINUE
      DO 1 I=1,N
      II=I
      DO 2 J=II,N
      IF(Y(I).GT.Y(J))GOTO3
      GOTO2
3     TEMP=Y(I)
      Y(I)=Y(J)
      Y(J)=TEMP
      TEMP=Y(I)
      Y(I)=[illegible]
      Y(J)=[illegible]
      [illegible]
      [illegible]
      [illegible]
[illegible]
[illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]=(Y(1)-Y([illegible]))/(NLY-1)
      [illegible]
      IF([illegible].GT.X[illegible](N))GOTO6
      [illegible]=X(I)
[illegible]
      IF(X(I).LE.XMAX)GOTO4
      XMAX=X(I)
1     [illegible]
      DELX=(XMAX-XMIN)/(NLX-1)
      TYPE703,DELY,DELX
703   FORMAT(35X,'DELY=',E15.8,10X,'DELX=',E15.8/)
      JX=NLX/10+1
      JY=NLY/5+1
      DO 5 I=1,JY
5     [illegible]=Y[illegible]+DELY*10.*(I-1)
      [illegible] I=1,JY
[illegible]   [illegible]=Y([illegible])-DELY*5.*(I-1)
      [illegible]I=1,NLY
      [illegible](I)=I[illegible]
      [illegible](I)=I[illegible]
[illegible]   [illegible](I)=IP
      [illegible]I=2,NLX
      [illegible]=I-(I/10)*10
      IF([illegible].1)GOTO 9
      [illegible](I)=IP

      [illegible]
[illegible]   [illegible](I)=ID
7     CONTINUE
      [illegible](1)=IP
      [illegible](NLY)=IP
      [illegible](1)=IP
      [illegible](NLY)=IP
      [illegible](1)=IP
      [illegible](1)=IP
      [illegible](NLX)=IP
```

```
      TYPE 92,(SX(I),I=1,JX)
      TYPE 91,(ILL(I),I=1,NLX)
92    FORMAT(15X,11(F9.3,1X))
91    FORMAT(20X,102A1)
101   FORMAT('    8450')
      IF(XMIN*XMAX.GT.0)GOTO16
      NZZ=-XMIN/DELX
      KCHECK=1
16    N1=0
      Y(N+1)=Y(N)-DELY
      DO10I=1,N
      NX=(X(I)-XMIN)/DELX+1
      NY=(Y(1)-Y(I))/DELY+0.5
      NY1=(Y(1)-Y(I+1))/DELY+0.5
      NK=NY1-NY
      IF(NK-1)61,62,63
61    ILC(NX)=ISYMB(IMY(I))
      GOTO10
63    NK=NK-1
      ILC(NX)=ISYMB(IMY(I))
      NP1=NY-(NY/5)*5
      NP2=NY/5+1
      IF(KCHECK.NE.1)GOTO300
      ILC(NZZ)=IZ
300   IF([illegible])GOTO[illegible]3
      IF([illegible].[illegible].[illegible].[illegible].0)GOTO302
      [illegible]=1,[illegible]
      [illegible].[illegible].[illegible])GOTO[illegible]
      [illegible]=
[illegible] [illegible]
[illegible] [illegible],[illegible]),(ILC(KX),KX=1,NLX)
      [illegible]
[illegible] [illegible],([illegible]),KL=1,NLX)
C     ****************************************
[illegible] [illegible]=1,[illegible]
      [illegible]=([illegible]+[illegible])/5.+1
      [illegible]=[illegible]+[illegible]-([illegible]P1-1)*5
      IF([illegible].NE.0)GOTO71
      IF(KCHECK.NE.1)GOTO307
      [illegible](NZZ)=IZ
307   TYPE 95,SY(NP1),(ILD(KL),KL=1,NLX)
      GOTO65
71    TYPE 97,(ILB(KL),KL=1,NLX)
65    CONTINUE
97    FORMAT(20X,102A1)
      CONTINUE
C     ****************************************
62    CONTINUE
      I[illegible]C([illegible])=ISY[illegible]B(IMY(I))
      [illegible]=[illegible]-([illegible]Y/5)*5
      [illegible]=[illegible]Y/5+1
      IF([illegible].[illegible].1)GOTO72
      I[illegible]C([illegible])=IZ
72    IF([illegible].[illegible]E.0)GOTO73
      IF([illegible].EQ.[illegible]Y-1.OR.NY.EQ.0)GOTO83
      DO 74 I[illegible]T=1,NLX
      IF(ILC([illegible]).NE.IB)GOTO74
      I[illegible]C([illegible])=IP
74    CONTINUE
83    TYPE 96,SY([illegible]P2),(ILC(KX),KX=1,NLX)
      GOTO105
73    TYPE 97,(ILC(KL),KL=1,NLX)
96    FORMAT(5X,F15.8,102A1)
C     ****************************************
105   CONTINUE
      DO66KL=1,[illegible]
66    ILC(KL)=IB
      ILC(1)=IP
      ILC([illegible]X)=IP
```

```
1         CONTINUE
          TYPE91,(TLL(I),I=1,NLX)
          TYPE92,(SX(I),I=1,JX)
          DO 79 JIK=1,N
79        Y(JIK)=XXX(JIK)
          RETURN
          END
```

EFFECT OF SHARPNESS INDEX ON DISTRIBUTION CURVE - " F(D)=1./(+D**N) "

N=	1.000	5.000	10.000	50.000	100.000	200.000	500.0
D(I)	FD(I)-CALCULATED FOR THE ABOVE SHARPNESS INDEX N						
0.500	66.667	96.970	99.902	100.000	100.000	100.000	100.
0.600	62.500	92.785	99.399	100.000	100.000	100.000	100.
0.700	58.824	85.611	97.253	100.000	100.000	100.000	100.
0.750	57.143	80.821	94.669	100.000	100.000	100.000	100.
0.770	56.497	78.698	93.173	100.000	100.000	100.000	100.
0.790	55.866	76.470	91.351	99.999	100.000	100.000	100.
0.810	55.249	74.147	89.160	99.997	100.000	100.000	100.
0.830	54.645	71.741	86.568	99.991	100.000	100.000	100.
0.850	54.054	69.266	83.551	99.970	100.000	100.000	100.
0.870	53.476	66.737	80.101	99.905	100.000	100.000	100.
0.890	52.910	64.168	76.230	99.706	99.999	100.000	100.
0.900	52.632	62.874	74.147	99.487	99.997	100.000	100.
0.910	52.356	61.575	71.973	99.112	99.992	100.000	100.
0.920	52.083	60.274	69.716	98.477	99.976	100.000	100.
0.930	51.813	58.973	67.386	97.413	99.930	100.000	100.
0.940	51.546	57.673	64.994	95.664	99.795	100.000	100.
0.950	51.282	56.377	62.549	92.855	99.411	99.996	100.
0.960	51.020	55.085	60.066	88.505	98.341	99.972	100.
0.970	50.761	53.800	57.556	82.097	95.461	99.774	100.
0.980	50.505	52.523	55.034	73.305	88.291	98.272	99.
1.000	50.000	50.000	50.000	50.000	50.000	50.000	50.
1.050	48.780	43.931	38.039	8.021	0.755	0.006	0.
1.100	47.619	38.307	27.826	0.845	0.007	0.000	0.
1.150	46.512	33.208	19.819	0.092	0.000	0.000	0.
1.200	45.455	28.667	13.905	0.011	0.000	0.000	0.
1.250	44.444	24.681	9.696	0.001	0.000	0.000	0.
1.300	43.478	21.218	6.763	0.000	0.000	0.000	0.
1.350	42.553	18.235	4.738	0.000	0.000	0.000	0.
1.400	41.667	15.678	3.342	0.000	0.000	0.000	0.
1.450	40.816	13.496	2.376	0.000	0.000	0.000	0.
1.500	40.000	11.636	1.705	0.000	0.000	0.000	0.
1.600	38.462	8.706	0.901	0.000	0.000	0.000	0.
1.700	37.037	6.580	0.494	0.000	0.000	0.000	0.
1.800	35.714	5.026	0.279	0.000	0.000	0.000	0.
1.900	34.483	3.882	0.163	0.000	0.000	0.000	0.
2.000	33.333	3.030	0.098	0.000	0.000	0.000	0.

APPENDIX -3

EFFECT OF SHARPNESS INDEX AND LIMITING REDUCED DENSITY ON DISTRIBUT
F(D)=1/(1+((D-DL)/(1-DL))**N)

SHARPNESS INDEX N IS : 2.0

MRDL:	0.50	0.60	0.70	0.80	0.90
D(I)	FD(I)-CALCULATED FOR THE ABOVE VALUES OF N AND DL				
0.500	100.000	94.118	69.231	30.769	5.882
0.600	96.154	100.000	90.000	50.000	10.000
0.700	86.207	94.118	100.000	80.000	20.000
0.750	80.000	87.671	97.297	94.118	30.769
0.770	77.423	84.701	94.937	97.800	37.175
0.790	74.828	81.591	91.743	99.751	45.249
0.810	72.233	78.393	88.149	99.751	55.249
0.830	69.657	75.153	84.191	97.800	67.114
0.850	67.114	71.910	80.000	94.118	80.000
0.870	64.616	68.699	75.694	89.087	91.743
0.890	[illegible]	65.547	71.372	83.160	99.010
0.910	[illegible]	62.476	67.114	76.775	99.010
0.930	[illegible]	59.502	62.961	70.299	91.743
0.940	[illegible]	58.055	60.976	67.114	86.207
0.950	55.249	56.637	59.016	64.000	80.000
0.960	54.159	55.249	57.107	60.976	73.529
0.970	53.090	53.990	55.249	58.055	67.114
0.980	52.040	52.562	53.444	55.249	60.976
0.990	51.010	51.265	51.694	52.562	55.249
1.000	50.000	50.000	50.000	50.000	50.000
1.050	45.249	44.138	42.353	39.024	30.769
1.1	40.984	39.024	36.000	30.769	20.000
1.150	37.175	34.595	30.769	24.615	13.793
1.200	33.784	30.769	26.471	20.000	10.000
1.25	30.769	27.469	22.930	16.495	7.547
1.300	28.090	24.615	20.000	13.793	5.882
1.350	25.707	22.145	17.561	11.679	4.706
1.400	23.585	20.000	15.517	10.000	3.846
1.450	21.692	18.130	13.793	8.649	3.200
1.500	20.000	16.495	12.329	7.547	2.703
1.600	17.123	13.793	10.000	5.882	2.000
1.700	14.793	11.679	8.257	4.706	1.538
1.800	12.887	10.000	6.923	3.846	1.220
1.900	11.312	8.649	5.882	3.200	0.990
2.000	10.000	7.547	5.056	2.703	0.820

APPENDIX -3

SHARPNESS INDEX N IS : 4.0

MRDI:	0.50	0.60	0.70	0.80	0.90
D(I)	FD(I)-CALCULATED FOR THE ABOVE VALUES OF N AND DI				
0.500	100.000	99.611	83.505	16.495	0.389
0.600	99.840	100.000	98.780	50.000	1.220
0.700	97.504	99.611	100.000	94.118	5.882
0.750	94.118	98.061	99.923	99.611	16.495
0.770	92.163	96.841	99.704	99.949	25.933
0.790	89.834	95.156	99.197	99.999	40.583
0.810	87.126	92.939	98.225	99.999	60.383
0.830	84.051	90.146	96.594	99.949	80.639
0.850	80.639	86.761	94.118	99.611	94.118
0.870	76.931	82.809	90.653	98.522	99.197
0.89[illegible]	[illegible]	78.353	86.141	96.061	99.990
[illegible]	[illegible]	73.169	80.639	91.617	99.990
[illegible]	[illegible]	68.341	74.323	84.853	99.197
[illegible]	[illegible]	65.703	70.942	80.639	97.504
[illegible]	[illegible]	63.044	67.465	75.964	94.118
[illegible]	[illegible]	60.383	63.932	70.942	88.527
0.[illegible]	56.156	57.734	60.383	65.703	80.639
0.9[illegible]	54.073	55.111	56.856	60.383	70.942
0.[illegible]	52.019	52.530	53.385	55.111	60.383
1.000	50.000	50.000	50.000	50.000	50.000
1.050	40.583	38.435	35.055	29.058	16.495
1.100	32.535	29.058	24.036	16.495	5.882
1.150	25.933	21.860	16.495	9.635	2.496
1.200	20.654	16.495	11.473	5.882	1.220
1.250	16.495	12.542	8.132	3.755	0.662
1.300	13.239	9.635	5.882	2.496	0.389
1.350	10.693	7.485	4.341	1.718	0.243
1.400	8.697	5.882	3.263	1.220	0.160
1.450	7.127	4.675	2.496	0.888	0.109
1.500	5.882	3.755	1.939	0.662	0.077
1.600	4.094	2.496	1.220	0.389	0.042
1.700	2.926	1.718	0.803	0.243	0.024
1.800	2.141	1.220	0.550	0.160	0.015
1.900	1.601	0.888	0.389	0.109	0.010
2.000	1.220	0.662	0.283	0.077	0.007

APPENDIX -3

SHARPNESS INDEX N IS : 6.0

MEDI:	0.50	0.60	0.70	0.80	0.90
D(I)	FD(I)-CALCULATED FOR THE ABOVE VALUES OF M AND DI				
0.500	100.000	99.976	91.929	8.071	0.0[illegible]4
0.600	99.994	100.000	99.863	50.000	0.137
0.700	99.592	99.976	100.000	99.462	1.538
0.750	98.462	99.723	99.998	99.976	8.071
0.770	97.581	99.414	99.984	99.999	17.162
0.790	96.333	98.864	99.927	100.000	36.081
0.810	94.625	97.949	99.758	100.000	65.298
0.830	92.366	96.512	99.342	99.999	89.474
0.850	89.474	94.375	98.462	99.976	98.462
0.870	[illegible]	91.359	96.795	99.817	99.927
0.8[illegible]	[illegible]	87.312	93.938	99.176	100.000
[illegible]	[illegible]	[illegible]	89.474	97.306	100.000
[illegible]	[illegible]	76.029	83.121	92.987	99.927
[illegible]	[illegible]	72.614	79.230	89.474	99.592
[illegible]	[illegible]	69.023	74.912	84.891	98.462
[illegible]	[illegible]	65.298	70.237	79.230	95.542
[illegible]	[illegible]	61.486	65.298	72.614	89.474
[illegible]	[illegible]	57.634	60.204	65.298	79.230
0.9[illegible]	53.127	53.790	55.068	57.634	65.298
1.000	50.000	50.000	50.000	50.000	50.00
1.050	36.081	33.033	28.396	20.770	8.07
1.10	25.088	20.770	15.109	8.071	1.538
1.15	17.162	12.890	9.071	3.364	0.408
1.200	11.724	8.071	4.458	1.538	0.137
1.250	8.071	5.151	2.566	0.765	0.054
1.300	5.625	3.364	1.538	0.408	0.024
1.350	3.978	2.250	0.957	0.231	0.012
1.400	2.856	1.538	0.616	0.137	0.006
1.450	2.081	1.074	0.408	0.085	0.004
1.500	1.538	0.765	0.277	0.054	0.002
1.600	0.874	0.408	0.137	0.024	0.001
1.700	0.521	0.231	0.073	0.012	0.000
1.800	0.323	0.137	0.041	0.005	0.000
1.900	[illegible]	0.085	0.024	0.004	0.000
[illegible]	[illegible]	[illegible]	0.015	0.002	[illegible]

APPENDIX -3

SHARPNESS INDEX N IS : 8.0

MPDL:	0.50	0.60	0.70	0.80	0.90
D(I)	FD(I)-CALCULATED FOR THE ABOVE VALUES OF N AND DI				
0.500	100.000	99.998	96.245	3.755	0.002
0.600	100.000	100.000	99.985	50.000	0.015
0.700	99.935	99.998	100.000	99.611	0.389
0.750	99.611	99.961	100.000	99.998	3.755
0.770	99.282	99.894	99.999	100.000	10.920
0.790	98.736	99.742	99.093	100.000	31.811
0.810	97.863	99.426	99.967	100.000	69.907
0.830	96.525	98.819	99.876	100.000	94.549
0.850	94.549	97.725	99.611	99.998	99.611
0.870	91.75[illegible]	95.869	98.948	99.977	99.993
0.89[illegible]	[illegible]	92.908	97.477	99.832	100.000
[illegible]	[illegible]	[illegible].185	94.549	99.170	100.000
[illegible]	[illegible]	[illegible].332	89.337	96.912	99.993
[illegible]	[illegible]	78.586	85.633	94.549	99.935
[illegible]	[illegible]	74.426	81.131	90.900	99.611
[illegible]	[illegible]	69.907	75.856	85.633	98.348
[illegible]	[illegible]2.129	65.106	69.907	78.586	94.549
[illegible]	58.093	60.117	63.459	69.907	85.633
[illegible]	54.032	55.046	56.739	60.117	69.907
1.000	50.000	50.000	50.000	50.000	50.000
1.050	31.811	28.014	22.562	14.367	3.755
1.100	18.869	14.367	9.100	3.755	0.389
1.150	10.920	7.259	3.755	1.124	0.065
1.200	[illegible].346	3.755	1.652	0.389	0.015
1.250	3.755	2.015	0.777	0.152	0.004
1.300	2.275	1.124	0.389	0.065	0.002
1.350	1.413	0.650	0.205	0.031	0.001
1.400	0.899	0.389	0.114	0.015	0.000
1.450	0.585	0.240	0.065	0.008	0.000
1.500	0.389	0.152	0.039	0.004	0.000
1.600	0.182	0.065	0.015	0.002	0.000
1.700	0.091	0.031	0.007	0.001	0.000
1.800	0.049	0.015	0.003	0.000	0.000
1.900	0.026	0.008	0.002	0.000	0.000
2.000	0.015	0.004	0.001	0.000	0.000

APPENDIX -3

SHARPNESS INDEX N IS : 10.0

PROB:	0.50	0.60	0.70	0.80	0.90
D(I)	FD(I)-CALCULATED FOR THE ABOVE VALUES OF N AND PROB				
0.500	100.000	100.000	98.295	1.705	0.000
0.600	100.000	100.000	99.998	50.000	0.002
0.700	99.990	100.000	100.000	99.902	0.098
0.750	99.902	99.995	100.000	100.000	1.705
0.770	99.790	99.981	100.000	100.000	6.763
0.790	99.571	99.942	99.999	100.000	27.926
0.810	99.168	99.841	99.996	100.000	74.147
0.830	98.456	99.606	99.977	100.000	97.253
0.850	97.253	99.099	99.902	100.000	99.392
0.870	[illegible]	98.074	99.660	99.997	99.999
0.890	[illegible]	96.143	98.972	99.965	100.000
[illegible]	[illegible]	92.750	97.253	99.747	100.000
[illegible]	[illegible]	87.255	93.444	98.672	99.999
[illegible]	[illegible]	83.551	90.304	97.253	99.990
[illegible]	[illegible]	79.172	86.095	94.669	99.902
[illegible]	69.716	74.147	80.706	90.304	99.399
[illegible]	64.994	68.560	74.147	83.551	97.253
[illegible]	59.066	62.549	66.595	74.147	90.304
[illegible]	55.034	56.296	58.395	62.549	74.147
1.000	50.000	50.000	50.000	50.000	50.000
1.050	27.326	23.544	17.632	9.696	1.705
1.100	13.905	9.696	5.331	1.705	0.098
1.150	6.763	3.975	1.705	0.370	0.010
1.200	2.742	1.705	0.601	0.098	0.002
1.250	1.705	0.773	0.233	0.030	0.000
1.300	0.901	0.370	0.098	0.010	0.000
1.350	0.494	0.186	0.044	0.004	0.000
1.400	0.279	0.098	0.021	0.002	0.000
1.450	0.163	0.053	0.010	0.001	0.000
1.500	0.098	0.030	0.005	0.000	0.000
1.600	0.038	0.010	0.002	0.000	0.000
1.700	0.016	0.004	0.001	0.000	0.000
1.800	0.007	0.002	0.000	0.000	0.000
1.900	0.003	0.001	0.000	0.000	0.000
[illegible]	0.002	0.000	0.000	0.000	0.000

APPENDIX -3

STEEPNESS INDEX N IS : 12.0

MDDI:	0.50	0.60	0.70	0.80	0.90
D(I)	FD(I)-CALCULATED FOR THE ABOVE VALUES OF [illegible] AND DI				
0.500	100.000	100.000	99.235	0.765	0.000
0.600	100.000	100.000	100.000	50.000	0.000
0.700	99.998	[illegible]	[illegible]	99.975	0.024
0.750	99.976	[illegible]	[illegible]	100.000	0.765
0.770	99.939	[illegible]	100.000	100.000	4.116
0.790	99.865	[illegible]	100.000	100.000	24.164
0.810	99.670	[illegible]	99.999	100.000	77.977
0.830	[illegible]	[illegible]	99.996	100.000	98.635
0.850	[illegible]	[illegible]	99.976	100.000	99.976
0.870	[illegible]	[illegible]	99.890	100.000	100.000
[illegible]	[illegible]	97.935	99.585	99.993	100.000
[illegible]	[illegible]	95.516	98.635	99.923	100.000
[illegible]	[illegible]	90.958	96.040	99.434	100.000
[illegible]	[illegible]	87.547	93.570	98.635	99.998
[illegible]	[illegible]	83.235	89.915	96.930	99.976
[illegible]	73.117	77.977	84.777	93.570	99.783
[illegible]	[illegible]	71.920	77.977	87.547	98.635
[illegible]	[illegible]	64.920	69.591	77.977	93.570
[illegible]	56.031	57.537	60.032	64.920	77.977
[illegible]	[illegible]	50.000	50.000	50.000	50.000
1.05	21.164	19.570	13.590	6.430	0.765
1.10	10.085	6.430	3.070	0.765	0.024
1.15	4.116	2.143	0.765	0.121	0.002
1.20	4.733	0.765	0.217	0.024	0.000
1.25	[illegible]	0.294	0.069	0.006	0.000
1.30	0.354	0.121	0.024	0.002	0.000
1.35	[illegible]	0.053	0.009	0.001	0.000
1.40	0.086	0.024	0.004	0.000	0.000
1.450	0.045	0.012	0.002	0.000	0.000
1.500	0.024	0.006	0.001	0.000	0.000
1.600	0.008	0.002	0.000	0.000	0.000
1.700	0.003	0.001	0.000	0.000	0.000
1.800	0.001	0.000	0.000	0.000	0.000
1.900	[illegible]	0.000	0.000	0.000	0.000
2.000	[illegible]	0.000	0.000	0.000	0.000

APPENDIX - 4

LISTING OF NUMERIC VALUES TO BE USED IN OUR MODEL

VESSEL FEED SIZE	DL	N	FU	FL	GPE
DENSE MEDIUM VESSEL					
6 X 4	0.9428	9.5	96.53	0.469	0.00985
4 X 2	0.8890	12.95	96.52	0.005	0.00985
2 X 1	0.8440	13.27	96.47	0.005	0.0135
1 X 1/2	0.7737	13.26	99.99	0.2E-05	0.0188
1/2 X 1/4	0.7500	12.3	96.99	3.03	0.02415
COMPOSITE	0.8045	12.59	99.99	1.63	0.01745
BAUM JIG					
6 X 3	0.5310	20.647	94.13	0.005	0.027
3 X 13/8	0.6730	13.60	99.6	5.17	0.0286
13/8 X 1/2	0.6302	9.27	99.995	2.27	0.0453
1/2 X 1/4	0.6670	4.42	99.42	5.39	0.0916
1/4 X 8	0.6819	3.73	99.99	4.03	0.101
8 X 14	0.6856	2.84	99.88	0.43	0.125
14 X 28	0.5879	2.796	99.52	15.05	0.241
COMPOSITE	0.8179	3.132	99.80	3.92	0.069
DENSE MEDIUM CYCLONE					
3/4 X 1/2	0.8730	15.23	99.88	0.248	0.0092
1/2 X 3/8	0.8730	13.55	98.18	0.295	0.0105
3/8 X 1/4	0.9010	10.88	97.67	0.1097	0.0103
1/4 X 8	0.8769	9.799	99.97	2.49	0.0143
8 X 14	[illegible]	9.[illegible]67	98.27	2.85	0.0219
14 X 28	.7593	10.[illegible]4	96.38	1.414	0.0283
COMPOSITE	[illegible].[illegible]213	12.744	99.99	2.89	0.016
W.O. CYCLONE					
1/4 X 4	.1135	8.259	98.24	0.005	0.0798
4 X 8	.135	9.29	97.63	0.005	0.0982
8 X 14	.2098	9.0	92.59	0.963	0.1081
14 X 28	0.11	10.22	92.68	0.065	0.106
28 X 48	.1505	9.42	92.93	0.005	0.109
48 X 100	0.4700	7.23	94.84	4.544	0.0918
100 X 200	0.5000	6.49	99.88	10.91	0.1033
COMPOSITE	0.6660	2.39	92.63	3.18	0.1823
CONCENTRATING TABLE					
3/8 X 1/4	0.7310	10.97	97.47	0.1E-05	0.02796
1/4 X 8	0.7513	6.87	99.39	0.005	0.04022
8 X 14	0.5700	8.81	99.99	1.02	0.04182
14 X 28	0.7450	4.56	99.99	0.998	0.06292
28 X 48	0.5119	7.28	98.77	2.31	0.07741
48 X 100	0.4575	6.53	98.10	0.005	0.0937
100 X 200	0.6059	4.69	99.99	18.445	0.151
COMPOSITE	0.7279	5.85	98.64	3.67	0.05503

FEED SIZE FRACTION IS IN INCHES AND MESH

DL - LIMITING REDUCED DENSITY

N - SHARPNESS INDEX

FU - MAXIMUM FEED REPORTING TO CLEAN COAL PRODUCT

FL - MINIMUM COAL REPORTING TO REFUSE

GPE - GENERALIZED PROBABLE ERROR

APPENDIX - 4

COMPARISION OF ABSOLUTE ERROR AND STANDARD DEVIATION BETWEEN GOTTFRIED EQUATION AND OUR MODEL

VESSEL	FEED SIZE FRACTION	ABSOLUTE ERROR GOTTFRIED	ABSOLUTE ERROR MODEL	STANDARD DEVIATION GOTTFRIED	STANDARD DEVIATION MODEL
DENSE MEDIUM VESSEL	6X4	0.15E+38	4.[illegible]	1.24E+19	0.6639
	4 X 2	3.893	1.[illegible]	0.424	0.4465
	2 X 1	1.5997	2.[illegible]	0.3813	0.4288
	1 X 1/2	14.244	4.[illegible]	1.136	0.6421
	1/2X1/4	4.2234	2.6[illegible]	0.849	0.434
	COMPOSITE	315.56	0.816	5.128	0.2608
BAUM JIG	6 X 3	328.7	14.61	4.846	1.02
	3X13/8	39.16	0.88	1.74	0.26
	13/8X1/2	83.655	0.356	2.44	0.1896
	1/2X1/4	10.773	1.049	0.796	0.248
	1/4X 8	288.13	0.9877	4.383	0.257
	8 X 14	12.896	0.708	0.803	0.188
	14X28	21.73	1.409	1.13	0.288
	COMPOSITE	136.03	1.644	2.676	0.294
DENSE MEDIUM CYCLONE	3/4X1/2	3.722	0.2744	0.516	0.14
	1/2X3/8	449.409	0.6529	5.686	0.216
	3/8X1/4	84.055	0.7138	2.543	0.234
	1/4X 8	1153.5 9	0.6704	8.7696	0.2114
	8 X 14	167.52	1.1396	3.236	0.2689
	14X28	20.526	3.0046	1.1698	0.4476
	COMPOSITE	835.822	0.6352	6.8143	0.188
HYDRO CYCLONE	1/4X 4	20.105	0.7394	1.057	0.2027
	4 X 8	252.37	0.8697	3.9716	0.2331
	8 X 14	21.094	9.533	1.054	0.7083
	14X28	81.715	10.664	2.131	0.7697
	28X48	130.94	8.737	2.7753	0.7169
	48X100	207.745	7.4458	3.3973	0.6432
	100X200	42.45	1.2097	1.536	0.2592
	COMPOSITE	5.841	1.3872	0.5544	0.2702
CONCENTRATING TABLE	3/8X1/4	2.573	1.131	0.4449	0.295
	1/4X 8	7.03	1.119	0.6623	0.2645
	8 X14	136.31	0.131	3.2381	0.1004
	14X28	104.55	0.437	2.5563	0.1652
	28X48	4.734	0.4745	0.5618	0.1779
	48X100	22.096	1.3687	1.14	0.2837
	100X200	44.143	6.062	1.611	0.5071
	COMPOSITE	15.119	3.92	0.9165	0.1476

TABLE - 5.3.10

DATA USED : DENSE MEDIUM VESSEL - COMPOSITE FEED (6" X 1/4" ...

USING GOTFRIED EQUATION

X(1)= 3.7838 X(2)= .17926000E-03 X(3)= 0.97920
X(4)= 0.9971 X(5)= .10710000E-01

D(I)	FD(I)	FCAL	DIFF
0.8050	100.00	37.35	62.65
0.8500	99.70	79.78	19.92
0.9000	99.10	98.99	0.11
0.9300	98.00	98.65	-0.65
0.9580	94.10	92.25	1.85
0.9700	90.10	84.61	5.49
1.0000	50.00	49.96	0.04
1.0270	17.90	16.88	1.02
1.0370	12.00	9.36	2.64
1.0620	5.00	1.87	3.13
1.0880	2.10	1.09	1.01
1.1400	0.70	1.07	-0.37
1.1750	0.30	1.07	-0.77

ABSOLUTE ERROR IN FULL RANGE = 0.31517986E+03
STANDARD DEVIATION IN FULL RANGE = 0.51249379E+01
ABSOLUTE ERROR WITH IN RANGE = 0.71783169E+01
STANDARD DEVIATION WITH IN RANGE = 0.11164167E+01

FD(I) - RANGE COVERED IS 1.071 % - 97.920 %

USING OUR MODEL

X(1)= 0.5045 X(2)= .12530000E+02 X(3)=99.99000
X(4)= 1.6268

D(I)	FD(I)	FCAL	DIFF
[illegible]	100.00	99.99	0.01
[illegible]	99.70	99.99	-0.29
[illegible]	99.10	99.98	-0.88
[illegible]	98.00	99.61	-1.61
[illegible]	94.10	95.38	-1.28
[illegible]	90.10	88.91	1.19
[illegible]	50.00	50.00	0.00
[illegible]	17.90	17.32	0.58
[illegible]	12.00	11.31	0.69
[illegible]	5.00	4.51	0.49
[illegible]	2.10	2.50	-0.40
[illegible]	0.70	1.73	-1.03
[illegible]	0.30	1.66	-1.36

ABSOLUTE ERROR IN FULL RANGE = 0.81633634E+00
STANDARD DEVIATION IN FULL RANGE = 0.26082183E+00
ABSOLUTE ERROR WITH IN RANGE = 0.77036728E+00
STANDARD DEVIATION WITH IN RANGE = 0.29256856E+00

FD(I) - RANGE COVERED IS 1.627 % - 99.990 %

APPENDIX - 5

DATA USED : BAUMJIG - 1/2 " X 1/4 " SIZE FEED

USING GOTTFRIED EQUATION.

X(1)= 1.7488 X(2)= .78043000E-01 X(3)= 0.88870
X(4)= 0.7998 X(5)= .88248000E-01

D(I)	FD(I)	FCAL	DIFF
0.7400	100.00	89.82	10.18
0.7630	99.70	94.24	5.46
0.7850	98.90	96.99	1.91
0.8110	97.10	97.26	-0.16
0.8350	94.50	94.49	0.01
0.8500	92.20	91.82	0.38
0.8650	89.20	88.59	0.61
1.0000	50.00	50.00	0.00
1.0880	27.50	29.57	-2.07
1.0980	25.00	27.80	-2.80
1.1090	22.90	25.97	-3.07
1.1450	19.70	20.92	-1.22
1.1880	16.90	16.50	0.40
1.2320	14.50	13.45	1.05
1.3060	11.50	10.63	0.87
1.4180	8.60	9.18	-0.58
1.5530	6.10	8.86	-2.76
1.7600	4.00	8.83	-4.83

ABSOLUTE ERROR IN FULL RANGE = 0.10772549E+02
STANDARD DEVIATION IN FULL RANGE = 0.79603979E+01
ABSOLUTE ERROR WITH IN RANGE = 0.31285733E+01
STANDARD DEVIATION WITH IN RANGE = 0.66853500E+00

FD(I) - RANGE COVERED IS 8.825 % - 88.879 %

USING THE MODEL

X(1)= 0.8670 X(2)= .14200000E+01 X(3)=99.42000
X(4)= 5.3900

D(I)	FD(I)	FCAL	DIFF
0.7400	100.00	99.29	0.71
0.7630	99.70	99.00	0.70
0.7850	98.90	98.37	0.53
0.8110	97.10	96.93	0.17
0.8350	94.50	94.62	-0.12
0.8500	92.20	92.57	-0.37
0.8650	89.20	90.00	-0.80
1.0000	50.00	50.00	0.00
1.0880	27.50	28.20	-0.70
1.0980	25.00	26.45	-1.45
1.1090	22.90	24.69	-1.79
1.1450	19.70	19.91	-0.21
1.1880	16.90	15.82	1.08
1.2320	14.50	12.93	1.57
1.3060	11.50	9.92	1.58
1.4180	8.60	7.67	0.93
1.5530	6.10	6.50	-0.40
1.7600	4.00	5.96	-1.96

ABSOLUTE ERROR IN FULL RANGE = 0.10493013E+01
STANDARD DEVIATION IN FULL RANGE = 0.24844236E+00
ABSOLUTE ERROR WITH IN RANGE = 0.93757667E+00
STANDARD DEVIATION WITH IN RANGE = 0.25872517E+00

FD(I) - RANGE COVERED IS 5.390 % - 99.420 %

TABLE - 5.3.7

DATA USED : BAUMJIG - 14 X 46 MESH SIZE

USING GOTFRIED EQUATION

X(1)= 2.4415 X(2)= .84770000E-01 X(3)= 0.68674
X(4)= 0.6079 X(5)= .29301000E+00

D(I)	FD(I)	FCAL	DIFF
0.5980	100.00	97.92	2.08
0.6120	99.80	97.97	1.83
0.6440	99.00	97.73	1.27
0.6820	97.50	96.58	0.92
0.7200	94.80	94.21	0.59
0.7400	92.50	92.43	0.07
0.7800	87.30	87.78	-0.48
0.8500	76.00	76.75	-0.75
1.0000	50.00	49.99	0.01
1.0460	42.20	43.55	-1.35
1.0640	40.00	41.41	-1.41
1.0940	37.60	38.35	-0.75
1.1460	34.30	34.41	-0.11
1.2420	29.80	30.72	-0.92
1.3380	26.70	29.59	-2.89
1.4900	22.90	29.31	-6.41
1.6700	19.30	29.30	-10.00
2.	14.30	29.30	-15.00

ABSOLUTE ERROR IN FULL RANGE = 0.21724075E+02
STANDARD DEVIATION IN FULL RANGE = 0.11304365E+01
ABSOLUTE ERROR WITH IN RANGE = 0.37181508E+00
STANDARD DEVIATION WITH IN RANGE = 0.41756798E+00

FD(I) - RANGE COVERED IS 29.301 % - 68.674 %

USING OUR MODEL

X(1)= 0.5879 X(2)= .27961300E+01 X(3)=99.52260
X(4)= 15.0469

D(I)	FD(I)	FCAL	DIFF
0.5980	100.00	99.52	0.48
0.6120	99.80	99.48	0.32
0.6440	99.00	99.07	-0.07
0.6820	97.50	97.64	-0.14
0.7200	94.80	94.83	-0.03
0.7400	92.50	92.74	-0.24
0.7[illegible]	87.30	87.39	-0.09
[illegible]	76.00	75.40	0.60
1.[illegible]	50.00	50.00	-0.00
1.0460	42.20	44.13	-1.93
1.0640	40.00	42.11	-2.11
1.0940	37.60	39.07	-1.47
1.1460	34.30	34.65	-0.35
1.2420	29.80	28.77	1.03
1.3380	26.70	24.91	1.79
1.4900	22.90	21.23	1.67
1.5780	19.30	18.80	0.50
2.0000	14.30	16.91	-2.61

ABSOLUTE ERROR IN FULL RANGE = 0.14087595E+01
STANDARD DEVIATION IN FULL RANGE = 0.28786838E+00
ABSOLUTE ERROR WITH IN RANGE = 0.12144255E+01
STANDARD DEVIATION WITH IN RANGE = 0.29452449E+00

FD(I) - RANGE COVERED IS 15.047 % - 99.523 %

TABLE - 5.3.5

DATA SET : DENSE MEDIUM CYCLONE - 8 X 14 MESH SIZE FEED

USING GOTFRIED EQUATION

X(1)= 2.6972 X(2)= .16777000E-02 X(3)= 0.97000
X(4)= 0.9145 X(5)= .18855000E-01

D(I)	FD(I)	FCAL	DIFF
0.8300	100.00	51.07	48.93
0.8600	99.70	80.67	19.03
0.8800	99.40	93.18	6.22
0.9010	98.60	98.42	0.18
0.9250	96.90	98.65	-1.75
0.9420	95.10	95.75	-0.65
0.9550	92.80	90.23	2.57
0.9580	92.00	88.50	3.50
1.0000	50.00	50.01	-0.01
1.0300	20.00	21.92	-1.92
1.0370	15.60	17.16	-1.56
1.0570	10.00	7.90	2.10
1.0750	7.30	3.99	3.31
1.1020	5.00	2.17	2.83
1.1400	2.90	1.89	1.01
1.1750	1.60	1.89	-0.29
1.2220	0.60	1.89	-1.29

ABSOLUTE ERROR IN FULL RANGE = 0.16760241E+03
STANDARD DEVIATION IN FULL RANGE = 0.32365338E+01
ABSOLUTE ERROR WITH IN RANGE = 0.47992628E+01
STANDARD DEVIATION WITH IN RANGE = 0.69276711E+00

FD(I) - RANGE COVERED IS 1.885 % - 97.000 %

USING OUR MODEL

X(1)= 0.3290 X(2)= .90670000E+01 X(3)=98.27000
X(4)= 2.8500

D(I)	FD(I)	FCAL	DIFF
0.8300	100.00	98.27	1.73
0.8600	99.70	98.27	1.43
0.8800	99.40	98.27	1.13
0.9010	98.60	98.23	0.37
0.9250	96.90	97.75	-0.85
0.9420	95.10	96.04	-0.94
0.9550	92.80	92.51	0.29
0.9580	92.00	91.24	0.76
1.0000	50.00	50.00	0.00
1.0300	20.00	20.41	-0.41
1.0370	15.60	16.39	-0.79
1.0570	10.00	9.25	0.75
1.0750	7.30	6.18	1.12
1.1020	5.00	4.17	0.83
1.1400	2.90	3.26	-0.36
1.1750	1.60	3.01	-1.41
1.2220	0.60	2.90	-2.30

ABSOLUTE ERROR IN FULL RANGE = 0.11420280E+01
STANDARD DEVIATION IN FULL RANGE = 0.26716428E+00
ABSOLUTE ERROR WITH IN RANGE = 0.51786109E+00
STANDARD DEVIATION WITH IN RANGE = 0.22756561E+00

FD(I) - RANGE COVERED IS 2.850 % - 98.270 %

TABLE - 5.3.6

DATA USED : DENSE MEDIUM CYCLONE - 14 X 28 MESH SIZE

USING GOTFRIED EQUATION

X(1)= 3.2237 X(2)= .25115000E-02 X(3)= [illegible].95759
X(4)= 0.8603 X(5)= .23930000E-01

D(I)	FD(I)	FCAL	DIFF
[illegible].7750	100.00	85.44	14.56
.8000	99.60	93.79	5.81
[illegible].8350	98.80	97.[illegible]	0.92
[illegible].8500	98.30	98.14	0.16
[illegible].9000	95.80	97.00	-1.20
0.9250	93.70	92.72	0.98
0.9400	91.70	87.79	3.91
0.9443	91.00	86.01	4.99
1.0000	50.00	50.00	0.00
1.0350	24.30	25.15	-0.85
1.0450	17.00	19.55	-2.55
1.0500	13.30	17.09	-3.79
1.0600	8.90	12.88	-3.98
1.09[illegible]	5.50	5.37	0.13
[illegible].105[illegible]	3.60	2.79	0.81
1.3[illegible]	1.40	2.39	-0.99

A[illegible] RANGE = 0.70525571E+02
S[illegible] RANGE = 0.11697741E+01
[illegible] RANGE = [illegible].79273723E+01
[illegible] RANGE = 0.93851966E+00

FD(I) - RANGE COVERED IS 2.393 % - 95.759 %

[illegible] MODEL

X(1)= [illegible].7580 X(2)= .10040000E+02 X(3)=96.38000
X(4)= [illegible].4140

D(I)	FD(I)	FCAL	DIFF
[illegible].775[illegible]	100.00	95.38	3.62
[illegible]	99.60	96.38	3.22
[illegible].835[illegible]	98.80	96.38	2.42
[illegible]	98.30	96.37	1.93
[illegible]	95.80	95.95	-0.15
[illegible]	93.70	94.24	-0.54
[illegible]	91.70	91.46	0.24
[illegible]	91.00	90.24	0.76
[illegible]	50.00	50.00	0.00
[illegible]	24.30	21.60	2.70
[illegible]	17.00	16.51	0.49
[illegible]	13.30	14.44	-1.14
[illegible]	8.90	11.08	-2.18
[illegible]	5.50	5.40	0.10
1.[illegible]	3.60	2.91	0.69
[illegible].3[illegible]	1.40	1.44	-0.04

ABSOLUTE ERROR IN FULL RANGE = 0.30045999E+01
STANDARD DEVIATION IN FULL RANGE = 0.44755632E+00
ABSOLUTE ERROR WITH IN RANGE = 0.13664018E+01
STANDARD DEVIATION WITH IN RANGE = 0.36964872E+00

FD(I) - RANGE COVERED IS 1.414 % - 96.380 %

TABLE - 5.3.3

DATA USED : HYDROCYCLONE - 14 X 28 MESH SIZE

USING GOTFRIED EQUATION

X(1)= 3.4993 X(2)= .90573000E-01 X(3)= 0.78954
X(4)= 0.5782 X(5)= .39170000E-01

D(I)	FD(I)	FCAL	DIFF
0.3000	100.00	73.56	26.44
0.3800	98.90	79.90	19.00
0.4800	96.80	82.61	14.19
0.5760	94.10	82.87	11.23
0.6180	92.50	82.86	9.64
0.6990	88.10	82.34	5.76
0.8000	80.60	78.51	2.09
0.8910	71.30	69.26	2.04
0.9400	64.00	61.55	2.45
0.9560	61.00	58.66	2.34
1.0000	50.00	50.00	0.00
1.0740	32.60	34.51	-1.91
1.1010	27.90	29.14	-1.24
1.1530	20.00	20.01	-0.01
1.1980	14.00	13.88	0.12
1.2800	6.30	7.14	-0.84
1.3220	4.10	5.48	-1.38
1.3600	3.00	4.66	-1.66
1.7340	1.00	3.92	-2.92

ABSOLUTE ERROR IN FULL RANGE = 0.81715391E+02
STANDARD DEVIATION IN FULL RANGE = 0.21306675E+01
ABSOLUTE ERROR WITH IN RANGE = 0.23443764E+01
STANDARD DEVIATION WITH IN RANGE = 0.51037855E+00

FD(I) - RANGE COVERED IS 3.917 % - 78.954 %

USING OUR MODEL

X(1)= 0.1100 X(2)= .10220000E+02 X(3)=92.68000
X(4)= 0.0650

D(I)	FD(I)	FCAL	DIFF
0.3000	100.00	92.68	7.32
0.3800	98.90	92.68	6.22
0.4800	96.80	92.67	4.13
0.5760	94.10	92.57	1.53
0.6180	92.50	92.42	0.08
0.6990	88.10	91.53	-3.43
0.8000	80.60	87.16	-6.56
0.8910	71.30	75.68	-4.38
0.9400	64.00	65.34	-1.34
0.9560	61.00	61.44	-0.44
1.0000	50.00	50.00	-0.00
1.0740	32.60	31.64	0.96
1.1010	27.90	26.05	1.85
1.1530	20.00	17.46	2.54
1.1980	14.00	12.16	1.84
1.2800	6.30	6.24	0.06
1.3220	4.10	4.46	-0.36
1.3600	3.00	3.31	-0.31
1.7340	1.00	0.22	0.78

ABSOLUTE ERROR IN FULL RANGE = 0.10663753E+02
STANDARD DEVIATION IN FULL RANGE = 0.76969520E+00
ABSOLUTE ERROR WITH IN RANGE = 0.50632425E+01
STANDARD DEVIATION WITH IN RANGE = 0.65809479E+00

FD(I) - RANGE COVERED IS 0.065 % - 92.680 %

[illegible] - 6.3.4

[illegible] HYDROCYCLONE - COMPOSITE(1/4'' X 200 MESH)

USING [illegible] EQUATION

X(1)= 1.7907 X(2)= .18142000E+02 X(3)= [illegible].78314
X(4)= 0.6742 X(5)= .15448000E+0[illegible]

X(I)	FD(I)	FCAL	DIFF
0.5850	93.60	93.57	0.03
0.7280	90.30	90.82	-0.52
0.7720	86.30	85.90	0.40
0.8230	81.20	79.02	2.18
0.8510	76.10	72.44	3.66
0.8935	71.00	67.03	3.97
1.0000	50.00	49.99	0.01
1.0400	45.00	44.36	0.64
1.0340	43.30	45.17	-1.87
1.0590	39.60	41.88	-2.28
1.1000	35.30	37.00	-1.70
1.1670	30.40	30.42	-0.02
1.2120	27.30	26.94	0.36
1.2550	24.40	24.23	0.17
1.2990	21.60	22.03	-0.43
1.3430	18.90	20.30	-1.40
[illegible]	16.30	18.95	-2.65
[illegible]	14.20	17.94	-3.74
[illegible]	12.60	17.22	-4.62
[illegible]	11.60	16.69	-5.09

[illegible] = 0.55433315E+01
[illegible] = 0.55456629E+00
[illegible] = 0.38789027E+01
[illegible] = 0.56853633E+00

[illegible] RANGE COVERED IS 15.448 % - 78.314 %

USING OUR MODEL

X(1)= 1.6650 X(2)= .23900000E+01 X(3)=92.63280
X(4)= 3.1785

X(I)	FD(I)	FCAL	DIFF
0.5850	93.60	92.55	1.05
0.7280	90.30	91.20	-0.90
0.7720	86.30	87.68	-1.38
[illegible]	81.20	81.43	-0.23
[illegible]	76.10	74.65	1.45
[illegible]	71.00	68.78	2.22
[illegible]	50.00	50.00	0.00
[illegible]	45.00	43.97	1.03
1.0340	43.30	44.82	-1.52
1.0590	39.60	41.35	-1.75
1.1000	35.30	36.28	-0.98
1.1670	30.40	29.49	0.91
1.2120	27.30	25.84	1.46
1.2550	24.40	22.91	1.49
1.2990	21.60	20.39	1.21
1.3430	18.90	18.27	0.63
1.3890	16.30	16.44	-0.14
1.4330	14.20	14.89	-0.69
1.4760	12.60	13.62	-1.02
1.5200	11.60	12.51	-0.91

ABSOLUTE ERROR IN FULL RANGE = 0.13871824E+01
STANDARD DEVIATION IN FULL RANGE = 0.27020289E+00
ABSOLUTE ERROR [illegible] RANGE = 0.14018015E+01
STANDARD DEVIATION [illegible] RANGE = 0.27906605E+00

TABLE - 5.3.7

[illegible] ISEP : CONCENTRATING TABLE - 48 X 100 MESH SIZE

USING GOTFRIED EQUATION

X(1)= 2.2230 X(2)= .63174000E-01 X(3)= 0.85370
X(4)= 0.7426 X(5)= .10650000E+00

D(I)	FD(I)	FCAL	DIFF
0.6530	100.00	91.30	8.70
0.7200	97.70	95.73	1.97
0.7300	94.10	95.13	-1.03
0.8200	90.30	91.57	-0.77
0.8620	86.20	84.84	1.36
0.8880	82.30	79.33	2.97
0.9040	78.00	75.53	2.47
1.0000	50.00	49.99	0.01
1.0260	42.00	43.35	-1.35
1.0400	37.50	39.99	-2.49
1.0790	30.00	31.60	-1.60
1.1250	23.70	23.83	-0.13
1.2200	10.00	14.66	-4.66
[illegible]	8.00	13.64	-5.64
[illegible]	6.00	12.53	-6.53
[illegible]	4.00	11.45	-7.45
[illegible]	3.00	10.90	-7.90
[illegible]	2.00	10.65	-8.65

[illegible] RANGE = 0.22087852E+02
[illegible] RANGE = 0.11338620E+01
[illegible] IN RANGE = 0.36453957E+01
[illegible] IN RANGE = 0.77946528E+00

D(I) - RANGE COVERED IS 10.650 % - 85.379 %

USING OUR MODEL

X(1)= 0.1575 X(2)= .65313900E+01 X(3)=98.10080
X(4)= 0.0050

D(I)	FD(I)	FCAL	DIFF
[illegible]	100.00	97.94	2.06
[illegible]	97.70	97.29	0.41
[illegible]	94.10	95.04	-0.94
[illegible]	90.80	91.77	-0.97
[illegible]	86.20	85.95	0.25
[illegible]	82.30	80.92	1.38
[illegible]	78.00	77.28	0.72
[illegible]	50.00	50.00	0.00
[illegible]	42.00	42.54	-0.54
[illegible]	37.50	38.76	-1.26
[illegible]	30.00	29.38	0.62
[illegible]	23.70	20.75	2.95
[illegible]	10.00	9.92	0.08
[illegible]	8.00	8.51	-0.51
[illegible]	6.00	6.78	-0.78
[illegible]	4.00	4.70	-0.70
[illegible]	3.00	3.08	-0.08
[illegible]	2.00	0.12	1.88

ABSOLUTE ERROR IN FULL RANGE = 0.13687461E+01
STANDARD DEVIATION IN FULL RANGE = 0.28375073E+00
ABSOLUTE ERROR WITH IN RANGE = 0.11984480E+01
STANDARD DEVIATION WITH IN RANGE = 0.27368413E+00

D(I) - RANGE COVERED IS 0.005 % - 98.101 %

03630

[redacted]ABLE - 5.3.8

DATA USED [redacted] 100 X 200 MESH SIZE

USING [redacted]RTED EQUATION

X(1)= 4.9932 X(2)= .26097000E-02 X(3)= 0.64672
X(4)= 0.6841 X(5)= .30829000E+00

D(I)	FD(I)	FCAL	DIFF
0.6060	100.00	95.43	4.57
0.6410	99.80	95.50	4.30
0.68[illegible]	99.20	95.50	3.70
0.7210	98.10	95.50	2.59
0.7600	96.90	95.44	1.46
0.8000	94.70	94.98	-0.28
0.8360	91.90	93.50	-1.60
0.8520	90.00	92.24	-2.24
0.9010	80.00	84.53	-4.53
1.0000	50.00	50.02	-0.02
1.0260	45.00	41.48	3.52
1.06[illegible]	40.00	31.41	5.59
1.1[illegible]	33.90	30.94	2.96
1.[illegible]	29.20	30.83	-1.63
1.3[illegible]	24.30	30.83	-6.53
1.[illegible]	21.90	30.83	-8.93
[illegible]	17.70	30.83	-13.13
[illegible]	12.00	30.83	-18.83

[illegible] RANGE = 0.44140594E+02
[illegible] = 0.16113676E+01
[illegible] = 0.13101297E+02
[illegible] = 0.20899926E+01

[illegible] RANGE COVERED IS 30.829 % - 61.672 %

USING OUR MODEL

X(1)= 0.6059 X(2)= .46925000E+01 X(3)=99.99420
X(4)= 18.4450

D(I)	FD(I)	FCAL	DIFF
[illegible]	100.00	99.99	0.01
[illegible]	99.80	99.99	-0.19
[illegible]	99.20	99.94	-0.74
[illegible].724[illegible]	98.10	99.54	-1.44
[illegible].7[illegible]	96.90	98.45	-1.55
[illegible]	94.70	95.59	-0.89
[illegible]	91.90	90.81	1.09
[illegible]	90.00	87.91	2.09
[illegible]	80.00	76.38	3.62
1.[illegible]	50.00	50.00	0.00
1.[illegible]	45.00	44.43	0.57
1.[illegible]	40.00	38.43	1.57
1.12[illegible]	33.90	30.58	3.32
1.2[illegible]	29.20	25.31	3.89
1.3[illegible]	24.30	21.49	2.81
1.4[illegible]	21.90	20.32	1.58
1.[illegible]	17.70	19.11	-1.41
[illegible]	12.00	18.58	-6.58

ABSOLUTE ERROR IN FULL RANGE = 0.60615262E+01
STANDARD DEVIATION IN FULL RANGE = 0.59712676E+00
ABSOLUTE ERROR WITH IN RANGE = 0.42533776E+01
STANDARD DEVIATION WITH IN RANGE = 0.55119206E+00

FD(I) - RANGE COVERED IS 18.445 % - 99.994 %